03/09/2021
Friday

MODULE : 05

ROTATION

KINEMATICS OF ROTATION

- Angular displacement , θ in radians
- Angular velocity , ω
- Angular accelaration , α

$$\omega = \frac{d\theta}{dt} \text{ rad/s} \qquad \alpha = \frac{d\omega}{dt} \text{ rad/s}^2$$

$$\omega = \frac{2\pi N}{60} \text{ rad/s}$$

N = angular velocity in rpm (revolutions per minute)

(a) For uniformly accelarated angular motion

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

(b) For uniform angular retardation

$$\omega = \omega_0 - \alpha t$$

$$\omega^2 = \omega_0^2 - 2\alpha\theta$$

$$\theta = \omega_0 t - \frac{1}{2}\alpha t^2$$

Relation between linear ~~accelaration~~ velocity and angular ~~accelaration~~ velocity

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega \qquad v = r\omega$$

Relation between linear accelaration and angular accelaration

$$a = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha \qquad a = r\alpha$$

(1) The armature of an electric motor, has angular speed of 1800 rpm at the instant when power is cut off. If it comes to rest in 6 seconds, calculate the angular deceleration assuming it is constant. How many revolutions does the armature make during this period?

$N_1 = 1800 \text{ rpm}$

$\omega_1 = \frac{2\pi N_1}{60} = 2\pi \times 30 \text{ rad/s}$

$\omega_2 = \omega_1 + \alpha t$

$0 = 2\pi \times 30 - \alpha \times 6$

$\alpha = \frac{2\pi \times 30}{6} = 31.4 \text{ rad}$

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(2) A body accelarates uniformly at 5 rad/s^2 & is found to be rotating at 90 rad/s at the end of 12 seconds. Detamine the initial velocity and angle turned during this interval.

$\alpha = 5 \text{ rad/s}^2$ $\quad \omega_2 = 90 \text{ rad/s}$ $\quad t = 12 s$

$\omega_2 = \omega_1 + \alpha t$

$90 = \omega_1 + 5 \times 12$

$\omega_1 = 90 - 60 = 30 \text{ rad/s}$

$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$= 30 \times 12 + \frac{1}{2} \times 5 \times 12 \times 12$

$= 360 + 360 = 720 \text{ rad}$

(3) During the starting phase of computer it is observed that a storage disc which was initially at rest executed 2.5 revolution in 0.5 s. Assuming that the angular accelaration of motion was uniform. Detamine

(a) α and (b) velocity of disc at $t = 0.5 s$

$\omega_1 = 0$

$\theta = 2.5 \text{ rev} = 2.5 \times 2\pi = 5\pi \text{ rad}$

$t = 0.5 s$

(i) angular accelaration

$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$5\pi = 0 + \frac{1}{2}\alpha \times 0.5 \times 0.5$

$\alpha = 125.66 \text{ rad/s}^2$

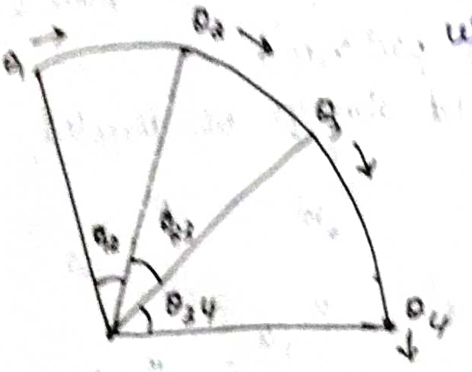
(ii) velocity of disc

$\omega_2 = \omega_1 + \alpha t$

$= 0 + 125.66 \times 0.5$

$= 62.83 \text{ rad/s}$

(5) A wheel accelarates, from rest, to a speed of 180 rpm uniformly in 0.4 s. It then rotates at that speed for 2 s before decelarating to rest in 0.3 s. Determine the total revolutions made by the wheel.



$\omega_0 = 0 \qquad N_2 = 180\ rpm$

$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 180}{60} = 6\pi\ rad/s$

$t_{12} = 0.4s \qquad t_{2-3} = 2s \qquad t_{3-4} = 0.3s$

$\omega_2 = \omega_1 + \alpha t_{1-2}$

$6\pi = 0 + \alpha \times 0.4 \Rightarrow \underline{\alpha = 47.1\ rad/s^2}$

$\theta_{1-2} = \omega_1 t_{1-2} + \frac{1}{2}\alpha t_{1-2}^2 = 0 + \frac{1}{2} \times 47.1 \times 0.4 \times 0.4 = \underline{3.77\ rad/s}$

During θ_{2-3} rotation of wheel, angular accelaration = 0

$\therefore \omega_3 = \omega_2$

$\theta_{2-3} = \omega_2 \times t_{2-3} = 6\pi \times 2 = 12\pi\ rad$

$\omega_4 = \omega_3 + \alpha \times t_{3-4} \Rightarrow 0 = 6\pi + \alpha \times 0.3 \Rightarrow \underline{\alpha = -62.8\ rad/s^2}$

$\theta_{3-4} = \omega_2 t_{3-4} + \frac{1}{2}\alpha t_{3-4}^2 = 6\pi \times 0.3 - \frac{1}{2} \times 62.8 \times 0.3^2$

$= \underline{2.83\ rad}$

Total angular displacement

$\theta = \theta_{1-2} + \theta_{2-3} + \theta_{3-4}$

$= 3.77 + 12\pi + 2.83 = 44.28\ rad$

$= \underline{7.05\ revolutions}$

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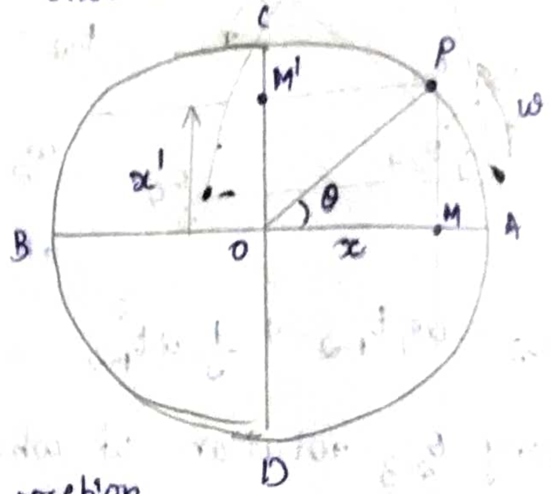
SIMPLE HARMONIC MOTION (SHM)

Conditions for a period motion to be simple harmonic.

- The accelaration of the body / particle performing periodic motion should be proportional to the distance of the body / particle from fixed point called the centre of SHM (mean position)
- The accelaration of the body / particle should always be directed towards the mean position.



$\theta = \omega t$; For one oscillation,

$t = t_p$ & $\theta = 2\pi$

$2\pi = \omega t_p$ $\boxed{t_p = \frac{2\pi}{\omega}}$

The displacement of M from mean position,

$OM = x = OP \cos\theta$ $\boxed{x = r\cos\omega t}$

$V = \frac{dx}{dt} = -r\omega \sin\omega t$

$\boxed{V = r\omega \sin\omega t} = r\omega \sin\theta = r\omega \frac{PM}{OP} = r\omega \frac{\sqrt{r^2 - x^2}}{r}$

$\boxed{V = \omega\sqrt{r^2 - x^2}}$

Accelaration of M,

$a = -r\omega^2 \cos\omega t$ $\boxed{a = -\omega^2 x}$

NOTES

- when the time of motion is measured from the mean position

$x = r\sin\omega t$

- when the time of motion is measured from the extreme position

$x = r\cos\omega t$

- In both cases, $\boxed{V = \omega\sqrt{r^2 - x^2}}$ $\boxed{a = -\omega^2 x}$

- Maximum velocity is at $x=0$ (at mean position)
- Maximum accelaration is at $x=r$ (at extreme position)

$$V_{max} = \omega\sqrt{r^2-0} \Rightarrow \boxed{V_{max} = r\omega} \quad \boxed{a_{max} = -\omega^2 r}$$

(5) A body moving with SHM has velocities of 10m/s and 4m/s at 2 & 4m distance from mean position. Find the amplitude & time period of the body.

$V = \omega\sqrt{r^2-x^2}$

At $x=2$, $V = 10$m/s

$10 = \omega\sqrt{r^2-2^2}$

At $x=4$m, $V = 4$m/s

$4 = \omega\sqrt{r^2-4^2}$

$$\frac{10}{4} = \frac{\sqrt{r^2-4^2}}{\sqrt{r^2-2^2}} \Rightarrow \frac{100}{16} = \frac{r^2-16}{r^2-4} \Rightarrow \underline{r = 4.28m}$$

$10 = \omega\sqrt{4.62^2-4^2}$

$\omega = \frac{10}{3.78} = \underline{2.64 \text{ rad/s}}$

$t_p = \frac{2\pi}{\omega} = \frac{2\times3.14}{2.64} = \underline{2.38s}$

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(6) A body is vibrating with SHM of amplitude 150mm and frequency 3cps. Calculate maximum velocity & accelaration of the body

$r = 150mm = 0.15m$

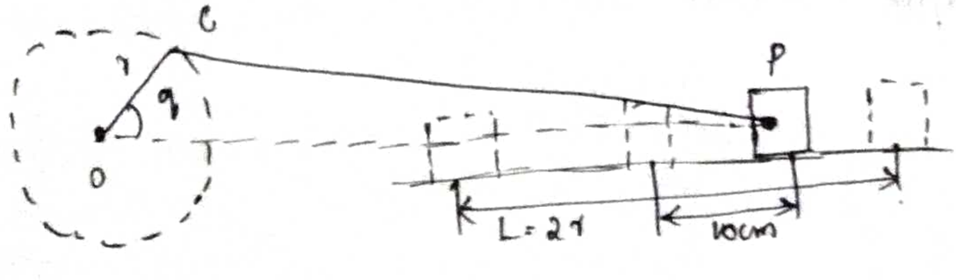
$f = 3cps$

$\omega = 2\pi f = 2\pi\times3 = 6\pi$ rad/s

$V_{max} = r\omega = 0.15\times6\pi = \underline{2.83 \text{m/s}}$

$a_{max} = \omega^2 r = 2.83\times6\pi = \underline{53.3 \text{m/s}^2}$

(7) The piston of an IC engine move with SHM. The crank rotates at 420 rpm and stroke length is 40cm. Find the v and a of piston when it is at a distance of 10cm from mean position.



Speed of crack = 420 rpm $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 420}{60} = 43.98$ rad/s

stroke length $L = 2 \times$ crank radius

crank radius $r = \frac{L}{2} = \frac{40}{2} = 20$ cm

$x = 10$ cm $= 0.1$ m

$$v = \omega\sqrt{r^2 - x^2} = \underline{7.62 \text{ m/s}}$$

$$a = \omega^2 r = \underline{193.42 \text{ m/s}^2}$$

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FREE VIBRATIONS

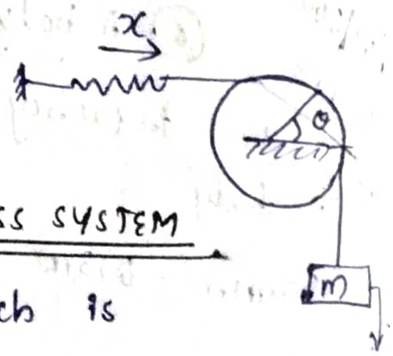
Free vibration :- If the disturbing force is applied just to start the motion and is then removed, leaving it to vibrate by itself

Force vibration :- If the disturbing force acts at periodic intervals on the system, the system is said to undergo forced vibration

DEGREE OF FREEDOM

- No. of independent co-ordinates required to defines the configuration of the system.

- Constraints to the motion reduce the degree of freedom



UNDAMPED FREE VIBRATIONS OF SPRING MASS SYSTEM

- The opposing force is called spring force which is proportional to the displacement of the spring.

- Spring force, $F \propto x$

$$\boxed{F = Kx}, \quad K = \text{stiffness of spring}$$

Unit = N/m

EQUATIONS OF FREE VIBRATIONS

$$\therefore \frac{d^2x}{dt^2} + \frac{K}{m}x = 0$$

$\boxed{\omega = \sqrt{K/m}}$

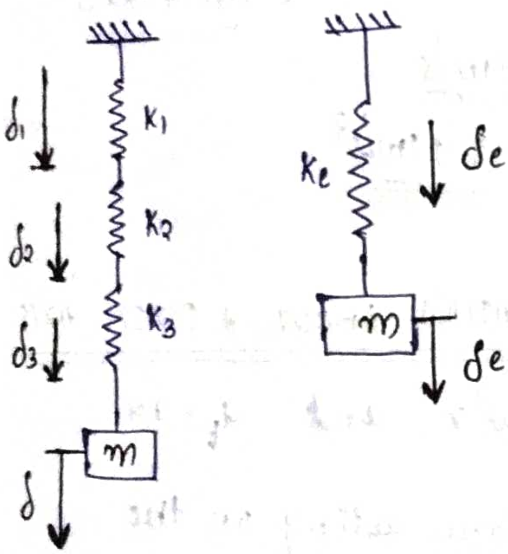
since the system vibrates freely, this frequency is called natural frequency & is denoted by ω.

$$\omega = 2\pi f, \quad f = \frac{\omega}{2\pi}$$

$$\boxed{f_n = \frac{1}{2\pi} \times \sqrt{K/m}}$$

SPRINGS

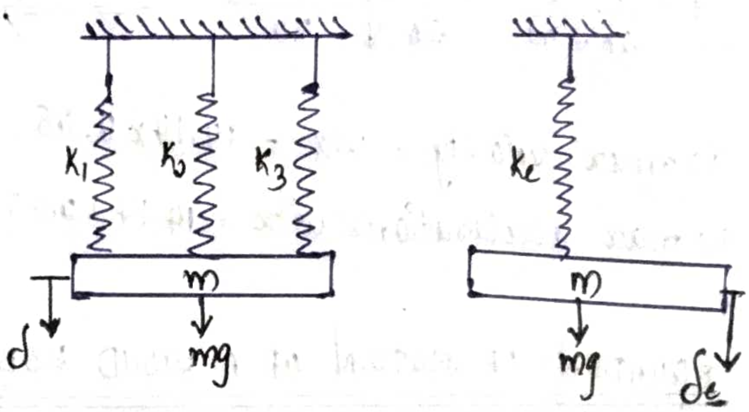
Springs in series



$$\delta_e = \delta = \delta_1 + \delta_2 + \delta_3$$

$$\boxed{\therefore \frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}}$$

Springs in Parallel

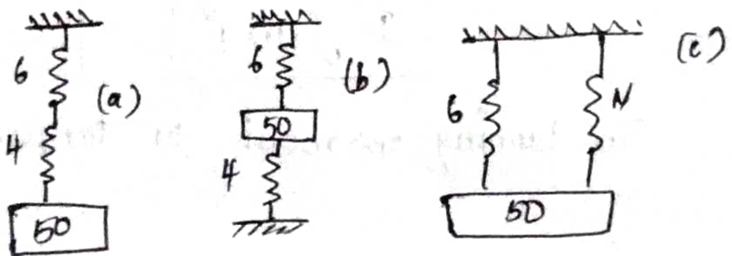


$$\delta e = \delta$$

$$\boxed{K_e = K_1 + K_2 + K_3}$$

(8) A body of mass 50kg is suspended by two springs of stiffness 4kN/m and 6kN/m as shown in fig(a), (b) and (c). The body is pulled 50mm down from its equilibrium position and then released. Calculate:-

(i) frequency of oscillation
(ii) max velocity
(iii) max acceleration



case 1 : Fig (a)

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} = \frac{1}{6} + \frac{1}{4} = \frac{10}{24} \Rightarrow K_e = \frac{24}{10} = 2.4\,kN/m$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi} \times \sqrt{\frac{2.4 \times 1000}{50}} = 1.10\,cps$$

$\omega = 2\pi f$
$= 2\pi \times 1.10 = 6.93$

(ii) max velocity $= \omega X = 6.93 \times 0.05 = 0.35\,m/s$

(iii) max accelaration $= \omega^2 x = 6.93 \times 0.35 = 2.4\,m/s^2$

case 2 :- Fig (b), Fig (c)

$K_e = K_1 + K_2 = 4 + 6 = 10\,kN/m$

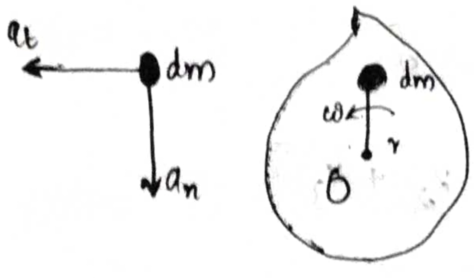
$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{10 \times 1000}{50}} = 2.25\,cps$$

$\omega = 2\pi f = 2\pi \times 2.25$
$= 14.14\,rad/s$

(ii) max velocity $= \omega X = 14.14 \times 0.05 = 0.707\,m/s$

(iii) max accelaration $= \omega^2 x = 14.14 \times 0.05 \times 14.14 = 9.99\,m/s^2$

EQUATION OF MOTION OF A RIGID BODY ROTATING ABOUT A FIXED AXI



$a_n = a_c = \omega^2 r$ and $a_t = r\alpha$

The tangential force acting on the elementary mass

= mass x tangential accelaration
$= dm \times (r\alpha)$

Moment of this force about O $= dm \times (r\alpha) \times r = dm\, r^2 \alpha$

The turning moment or torque on whole body about O,

$T = \int dm\, r^2 \alpha = I\alpha$. I = mass moment of inertia of body

$$\boxed{I = \int dm\, r^2}$$

The turning moment or torque $\boxed{T = I\alpha}$

KINETIC ENERGY DUE TO ROTATION

Kinetic energy of elementary mass $= \frac{1}{2} dm\, v^2 = \frac{1}{2} dm\, \omega^2 r^2 = \frac{1}{2} dm\, r^2 \omega^2$

Kinetic energy of whole body,

$$K.E = \int \frac{1}{2} dm\, r^2 \omega^2 = \frac{1}{2} I \omega^2$$

$$\boxed{K.E = \frac{1}{2} I \omega^2}$$

WORKDONE IN ROTATION

$$\boxed{\text{Workdone} = T \times \theta}$$

WORK-ENERGY EQUATION FOR ROTATION

The workdone by a torque acting on a body during an angular displacement is equal to the change in K.E of the body during the same displacement.

(9) A string 5m long is wound around the axle of a wheel. The string is pulled with a constant force of 250N. The wheel rotates at 800 rpm when the string leaves the axle. Find the moment of inertia of the wheel

$r\theta = 5$

$\theta = \frac{5}{r}$

Force $= 250N$

Length of string $= 5m$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 10\pi \text{ rad/s}$$

Change in K.E = work done

$$\left[\frac{1}{2} I \omega^2 - 0\right] = T\theta$$

$$= F \times r \times \frac{5}{r} = F \times 5 = 250 \times 5 = 1250 \text{ Nm}$$

$$\frac{1}{2} \times I \times (10\pi)^2 = 1250 \Rightarrow I = \frac{2500}{(10\pi)^2} = 2.53 \text{ kgm}^2$$