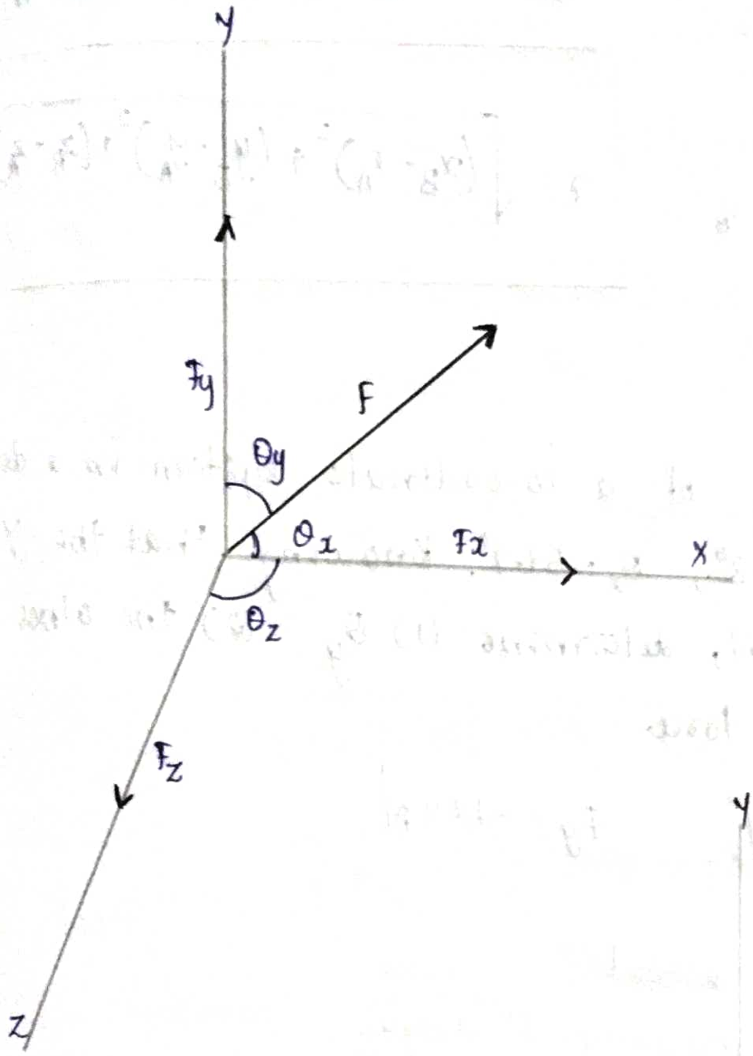
29/09/2021 Wednesday

MODULE : 03

FORCES IN SPACE

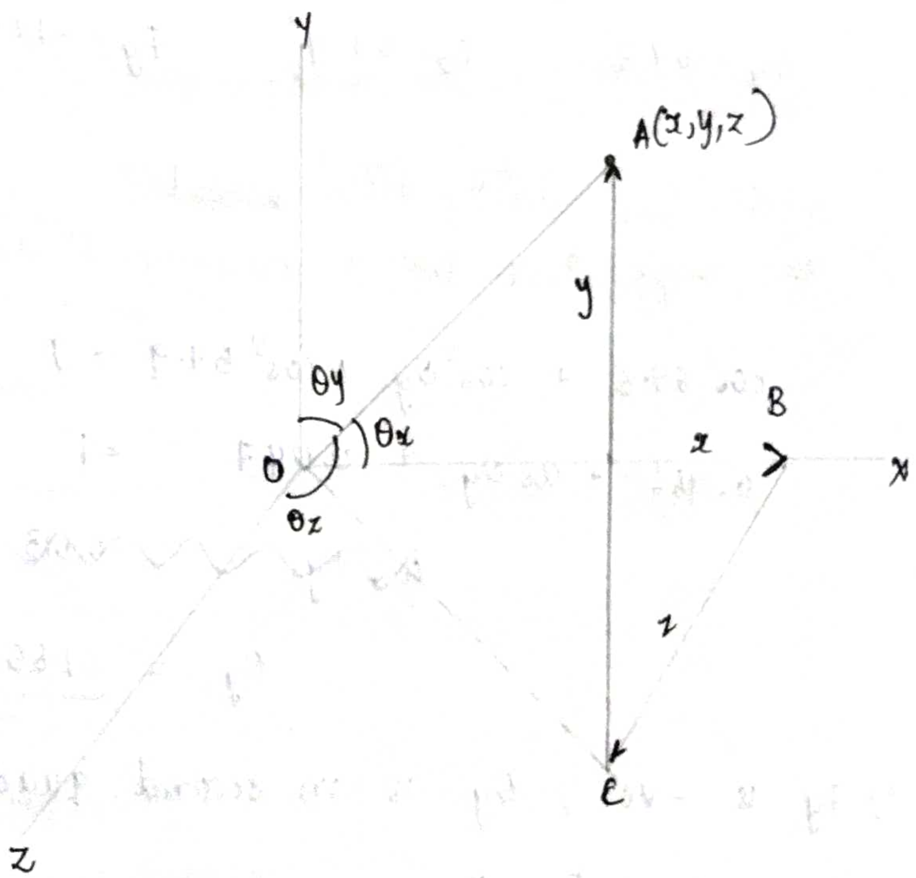


$$F_x = F\cos\theta_x$$

$$F_y = F\cos\theta_y$$

$$F_z = F\cos\theta_z$$

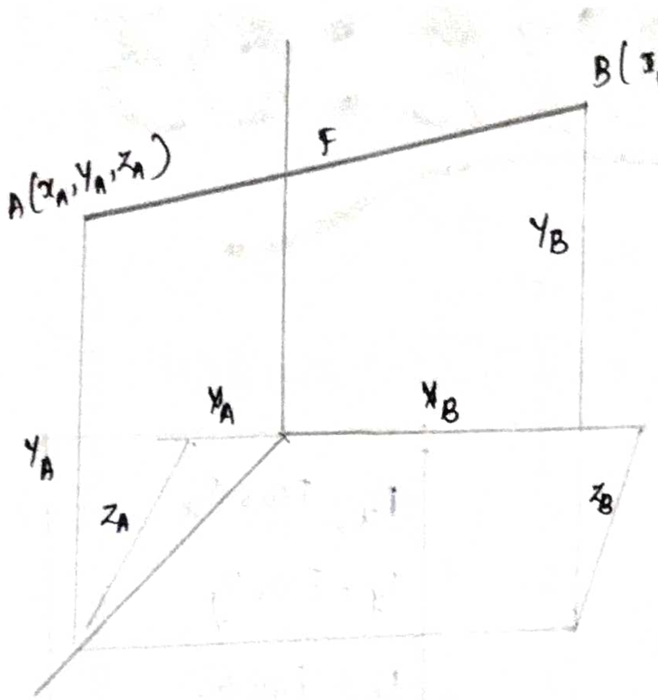
$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$



$$OA^2 = OC^2 + CA^2$$
$$= OB^2 + BC^2 + CA^2$$

$$r^2 = x^2 + y^2 + z^2$$

$$r = \sqrt{x^2 + y^2 + z^2}$$



$\vec{AB} = X_B - X_A$

$\vec{BA} = X_A - X_B$

$X = X_B - X_A$

$Y = Y_B - Y_A$

$Z = Z_B - Z_A$

$$\boxed{r = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

① A force acts at the origin of a co-ordinate system in a directi defined by the angle $\theta_x = 69.3°$, $\theta_z = 57.9°$. Knowing that the y component of force is -174 N, determine (1) θ_y (2) the other components & magnitude of force.

$\theta_x = 69.3°$ $\theta_z = 57.9°$ $F_y = -174$ N

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

The angle eq. b/w 3 mutually ⊥ angles

1.013

1.584

$\cos^2 69.3 + \cos^2\theta_y + \cos^2 57.9 = 1$

$0.966 + \cos^2\theta_y + 0.047 = 1$

$\cancel{\cos^2\theta_y}$ ~~0.013~~

180 − θ

$\theta_y = \underline{39.65 \text{ or } 140.35}$

∵ F_y is −ve, θ_y is in second quadrant & thus $\theta_y = 140.3$

$F_y = F\cos\theta_y$

$-174 = F\cos(140.35) \Rightarrow F = \dfrac{-174}{\cos(140.35)} = \underline{225.98}$

$F_x = F\cos\theta_x = 225.95 \times \cos 69.3 = 225.95 \times 0.983 = \underline{79.87}$

$F_z = F\cos\theta_z = 225.95 \times \cos 57.9 = 225.95 \times 0.218$
$= \underline{120N}$

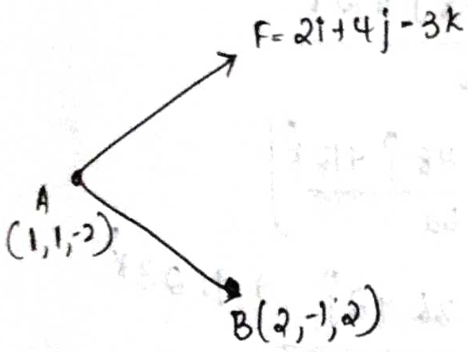
MOMENT

$M = F \times d$

$M = Fr\sin\theta \rightarrow \boxed{M = \vec{r} \times \vec{F}}$

$$\vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = i(yF_z - zF_y) - j(xF_z - zF_x) + k(xF_y - yF_x)$$

(2) A Force $F = 2i + 4j - 3k$ is applied at a point $A(1,1,-2)$. Find the moment of force F about $B(2,-1,2)$



$x_A = 1 \quad x_y = 1 \quad x_z = -2 \quad x_B = 2 \quad y_B = -1 \quad z_B = 2$

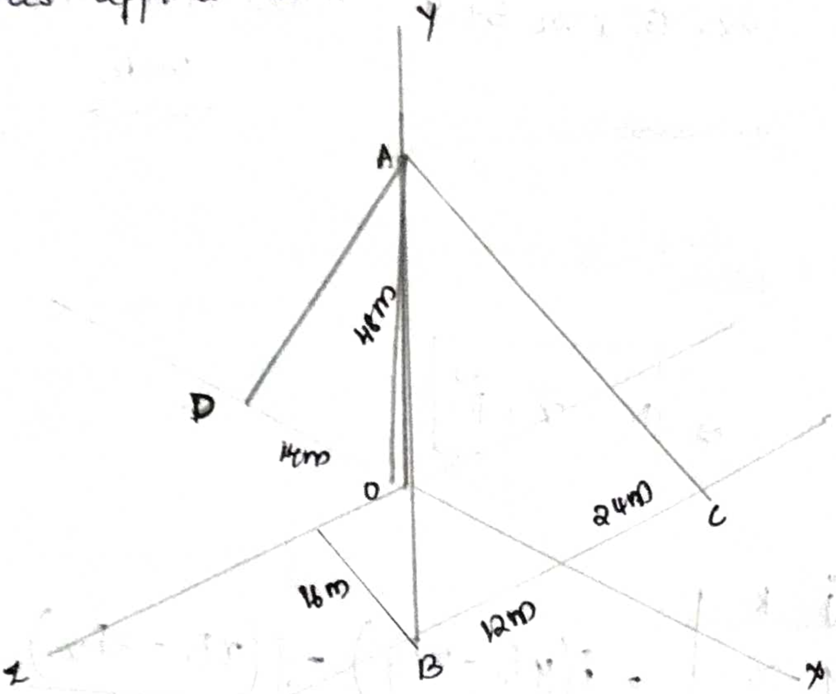
$r = (x_A - x_B)\hat{i} + (y_A - y_B)\hat{j} + (z_A - z_B)\hat{k}$
$= (1-2)\hat{i} + (1--1)\hat{j} + (-2-2)\hat{k}$
$= -1\hat{i} + 2\hat{j} - 4\hat{k}$

$$\vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ -1 & 2 & -4 \\ 2 & 4 & -3 \end{vmatrix} = \hat{i}(2\times -3 - -4\times 4) - \hat{j}(3 - -8) + \hat{k}(-4 - 4)$$

$= \hat{i}(-6+16) - \hat{j}(3+8) + \hat{k}(-4-4)$

$= \underline{10\hat{i} - 11\hat{j} - 8\hat{k}}$

30/09/2021 (3) A post is held vertical position by 3 cables AB, AC & AC. If T of cable AB = 40N, Calculate T of AC & AD so that resultant of 3 forces applied at A is vertical.



A (0, 48, 0)

B (16, 0, 12)

C (16, 0, 24)

D (-14, 0, 0)

$$r_{AB} = \sqrt{(16-0)^2 + (0-48)^2 + (12-0)^2} = \sqrt{16^2 + 48^2 + 12^2} = \sqrt{2704} = \underline{\underline{52\,m}}$$

$$r_{AC} = \sqrt{(16-0)^2 + (0-48)^2 + (24-0)^2} = \sqrt{3136} = \underline{\underline{56\,m}}$$

$$r_{AD} = \sqrt{(-14)^2 + (-48)^2 + 0^2} = \sqrt{2500} = \underline{\underline{50m}}$$

Unit vector in direction of AB = $\dfrac{16\hat{i} - 48\hat{j} + 12\hat{k}}{52}$

Force vector in direction of AB = $40\left[\dfrac{16\hat{i} - 48\hat{j} + 12\hat{k}}{52}\right]$

$$= 12.31\hat{i} - 36.92\hat{j} + 9.23\hat{k}$$

Unit vector in direction of AC = $\dfrac{16\hat{i} - 48\hat{j} + 24\hat{k}}{56}$

Force vector $= F_{AC}\left[\dfrac{16\hat{i} - 48\hat{j} + 24\hat{k}}{56}\right]$

$$= 0.29F_{AC}\hat{i} - 0.86F_{AC}\hat{j} - 0.43F_{AC}\hat{k}$$

Unit vector in direction of AD = $-14\hat{i} - 48\hat{j} / 50$

Force vector $= F_{AD}\left[\frac{-14\hat{i} - 48\hat{j}}{50}\right] = -0.28 F_{AD}\hat{i} - 0.96 F_{AD}\hat{j}$

Resultant force at A = $F_{AB} + F_{AC} + F_{AD}$

$= (12.31\hat{i} - 36.92\hat{j} + 9.23\hat{k}) + (0.29 F_{AC}\hat{i} - 0.89 F_{AC}\hat{j} - 0.43 F_{AC}\hat{k})$
$+ (-0.28 F_{AD}\hat{i} - 0.96 F_{AD}\hat{j})$

$= (12.31\hat{i} + 0.29 F_{AC}\hat{i} - 0.28 F_{AD}\hat{i}) + (-36.92 - 0.89 F_{AC} - 0.96 F_{AD})\hat{j}$
$+ (9.23\hat{k} - 0.43 F_{AC}\hat{k})$.

For resultant to vertical, x & z comp. = 0.

$F_z = 9.23 - 0.43 F_{AC} = 0$

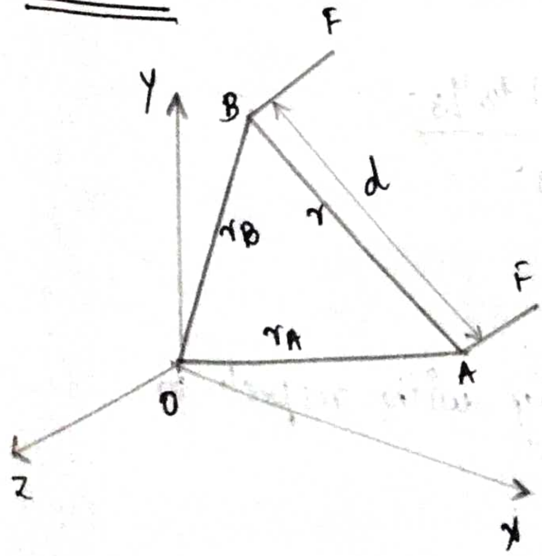
$F_{AC} = \frac{9.23}{0.43} = 21.47 N$ $\quad F_{AC} = 21.47 N$

$F_x = 12.31 + 0.29 F_{AC} - 0.28 F_{AD} = 0$

$12.31 + 0.29 \times 21.47 - 0.28 F_{AD} = 0$

$F_{AD} = \frac{18.5363}{0.28} = 66.20 N$ $\quad F_{AD} = 66.20 N$

COUPLE



$\boxed{M = \vec{r} \times \vec{F}}$

r = dis. b/w the points

4) Two forces $\vec{F_1} = 50i + 80j + 100k$ & $F_2 = -50i - 80j - 100k$ acts at point $A(0.7, 1.5, 1)$ & $B(1, 0.9, -1)$ respectively. Calculate the moment of the force & ⊥ʳ distance b/w the forces.

$\vec{F_1} = 50\hat{i} + 80j + 100k$

$\vec{F_2} = -50\hat{i} - 80j - 100k$

$x_A = 0.7 \quad x_B = 1.5 \quad x_C = 1$

$x_A = 1 \quad X_B = 0.9 \quad X_C = -1$

$|\vec{r}| = \sqrt{(1-0.7)^2 + (0.9-1.5)^2 + (-1-1)^2}$

$= \sqrt{(0.3)^2 + (-0.6)^2 + (-2)^2} = \sqrt{0.09 + 0.36 + 4}$

$= \sqrt{4.45} = \underline{\underline{2.11}}$

$\vec{r} = -0.3\hat{i} + 0.6\hat{j} + 2\hat{k}$

$$M = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ -0.3 & 0.6 & 2 \\ 50 & 80 & 100 \end{vmatrix} = (60-160)\hat{i} + (100+30)\hat{j} + (-24-30)$$

$$= \underline{100\hat{i} + 130\hat{j} - 54\hat{k}}$$

$|\vec{M}| = \sqrt{100^2 + 130^2 + 54^2} = 172.67$

Magnitude of force $F = \sqrt{50^2 + 80^2 + 100^2} = 137.48$

Moment of couple $= F \times d \Rightarrow d = \frac{M}{F} = \frac{172.67}{137.48} = \underline{1.26m}$

CENTROID OF COMPOSITE AREAS

$$\bar{x} = \frac{\int x a}{\int a} = \frac{\Sigma(x a)}{\Sigma a} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 \ldots}{a_1 + a_2 + a_3 \ldots}$$

$$\bar{y} = \frac{\int y a}{\int a} = \frac{\Sigma(y a)}{\Sigma a} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 \ldots}{a_1 + a_2 + a_3 \ldots}$$

5) Locate the centroid of 'T' section

Since the section is symmetrically with respect to y-axis, $\bar{x} = 0$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

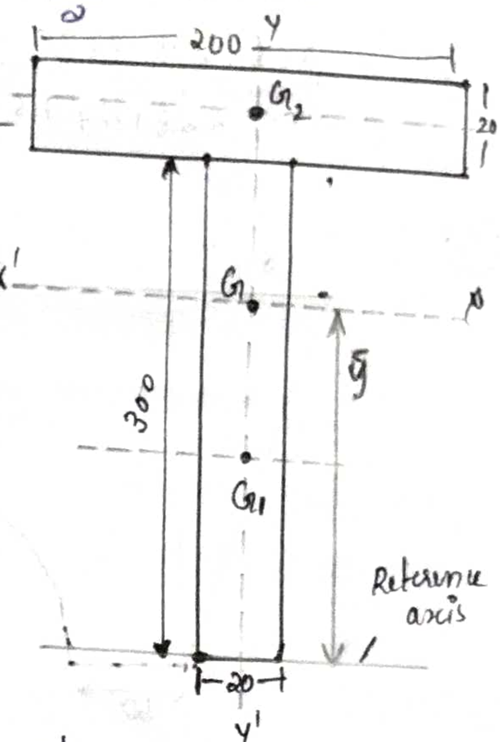
$a_1 = 300 \times 20 = 6000\,mm^2$

$a_2 = 200 \times 20 = 4000\,mm^2$

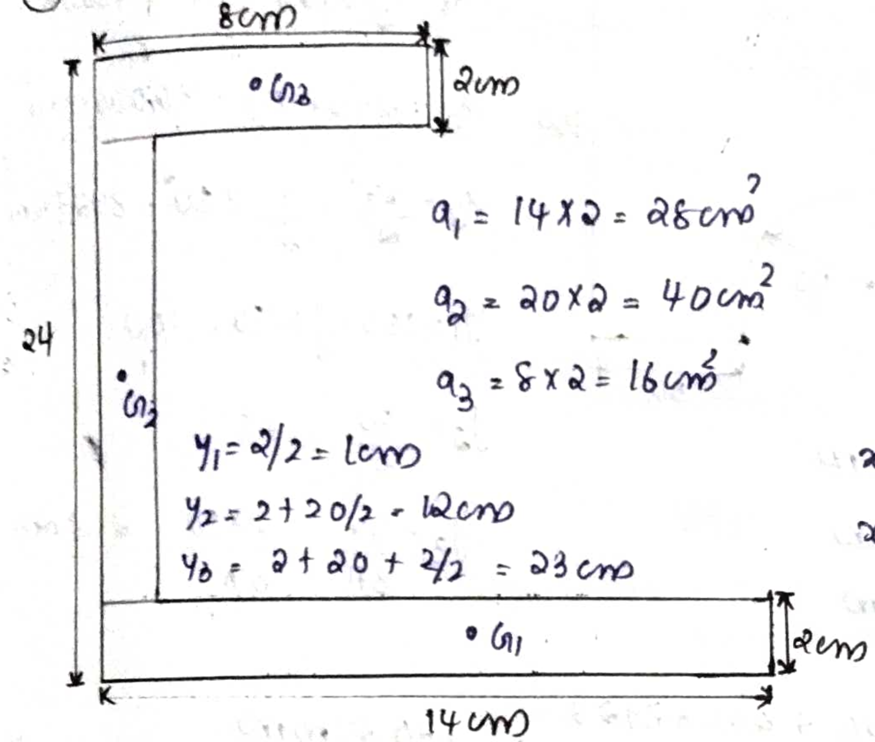
$y_1 = \frac{300}{2} = 150\,mm$

$y_2 = 300 + \frac{20}{2} = 310\,mm$

$\bar{y} = \frac{6000 \times 150 + 4000 \times 310}{6000 + 400} = \underline{214\,mm}$



(6) Locate the centroid of the area



$a_1 = 14 \times 2 = 28\,cm^2$

$a_2 = 20 \times 2 = 40\,cm^2$

$a_3 = 8 \times 2 = 16\,cm^2$

$y_1 = 2/2 = 1\,cm$

$y_2 = 2 + 20/2 = 12\,cm$

$y_3 = 2 + 20 + 2/2 = 23\,cm$

$x_1 = 14/2 = 7\,cm$

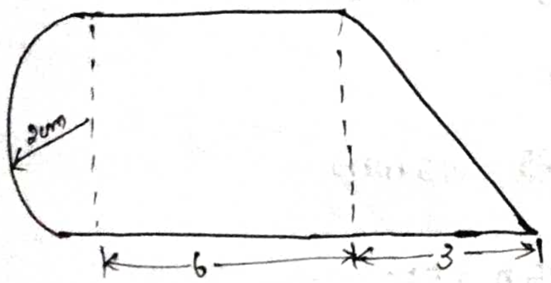
$x_2 = 2/2 = 1\,cm$

$x_3 = 8/2 = 4\,cm$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{28 \times 7 + 40 \times 1 + 16 \times 4}{28 + 40 + 16} = \underline{3.57\,cm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{28 \times 1 + 40 \times 12 \times 23 \times 4}{28 + 40 + 16} = \underline{10.43\,cm}$$

(7)



$a_1 = \frac{\pi r^2}{2} = \frac{\pi}{2} \times 2^2 = 6.28\,cm^2$

$a_2 = 6 \times 4 = 24\,cm^2$

$a_3 = \frac{1}{2} \times 3 \times 4 = 6\,cm^2$

$x_1 = r - \frac{4r}{3\pi} = 2 - \frac{4 \times 2}{3\pi} = 1.15\,cm$

$x_2 = 2 + 6/2 = 5\,cm$

$x_3 = 2 + 6 + \frac{1}{3} \times 3 = 9\,cm$

$y_1 = 2\,cm$

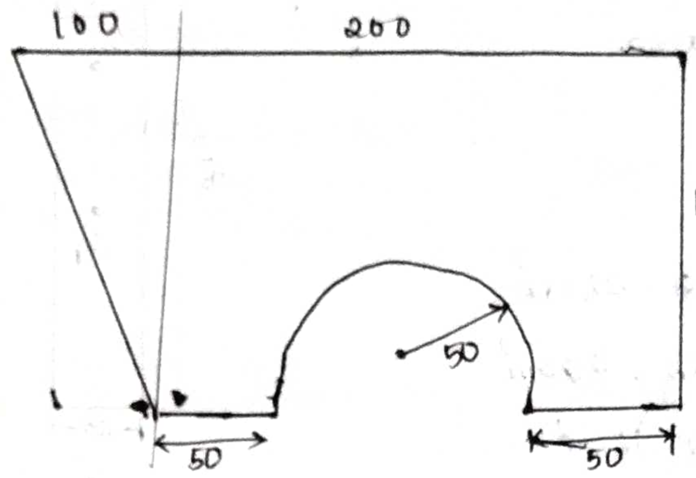
$y_2 = 2\,cm$

$y_3 = \frac{1}{3} \times 4 = 1.33\,cm$

$$\bar{x} = \frac{6.28 \times 1.15 + 24 \times 5 + 6 \times 9}{6.28 + 24 + 6} = 5 cm$$

$$\bar{y} = \frac{6.28 \times 2 + 24 \times 2 + 6 \times 1.33}{6.28 + 24 + 6} = 1.89 cm$$

(8)



$a_1 = \frac{1}{2} \times 100 \times 150 = 7500 mm^2$

$a_2 = 200 \times 150 = 30000 mm^2$

$a_3 = \frac{\pi r^2}{2} = \frac{\pi}{2} \times 50^2 = 3927 mm^2$

$y_1 = 150 - \frac{1}{3} \times 150 = 100$

$y_2 = \frac{150}{2} = 75$

$y_3 = \frac{4r}{3\pi} = \frac{4 \times 50}{3\pi} = 21.2 mm$

$x_1 = 100 - \frac{1}{3} \times 100 = 66.67 mm$

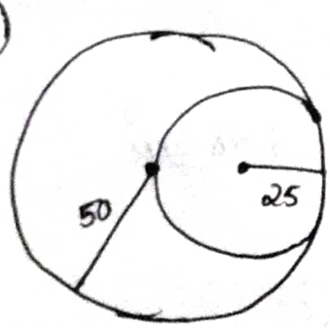
$x_2 = 100 + 200/2 = 200 mm$

$x_3 = 100 + 50 + 50 = 200 mm$

$$\bar{x} = \frac{66.7 \times 7500 + 200 \times 30000 + 200 \times 3927}{7500 + 30000 + 3927} = 170.21 mm$$

$$\bar{y} = \frac{100 \times 7500 + 75 \times 30000 + 21.2 \times 3927}{7600 + 30000 + 3927} = 86.88 mm$$

(9)



$a_1 = \pi R^2 = \pi \times 50 \times 50 = 2500\pi \; mm^2$

$a_2 = \pi r^2 = \pi \times 25^2 = 625\pi \; mm^2$

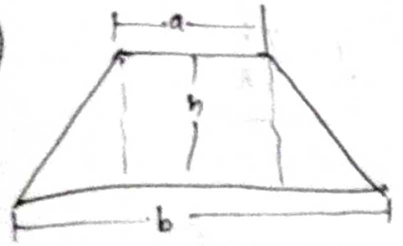
$x_1 = R = 50 mm$

$x_2 = R + r = 50 + 25 = 75 mm$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{2500\pi \times 50 - 625\pi \times 75}{2500\pi - 625\pi} = 41.67 mm$$

$\bar{y} = 0$

(10)



$a_1 = ab$ $\qquad y_1 = b/2$

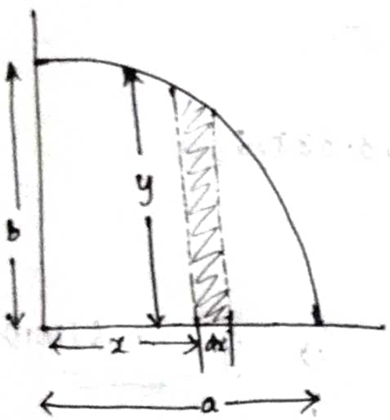
$a_2 = a_3 = \frac{1}{2}\frac{(b-a)}{2} h$ $\qquad y_2 = y_3 = \frac{1}{3}h$

$a_1 + a_2 + a_3 = \frac{a+b}{2} h$

$\bar{x} = 0$

$$\bar{y} = \frac{ab \times \frac{b}{2} + 2\left[\frac{1}{2}\left(\frac{b-a}{2}\right)h\right]}{\left(\frac{a+b}{2}\right)h} = \frac{2a+b}{a+b} \times \frac{h}{3}$$

(11) Quadrent eclipse



Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$

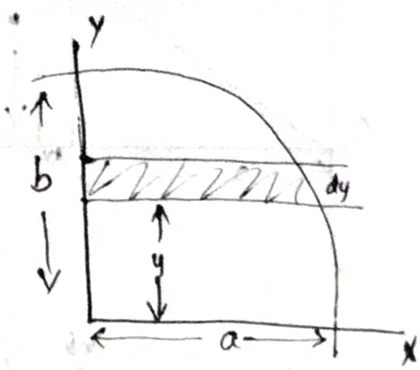
$y = b/a \sqrt{a^2 - x^2}$

$x = \frac{a}{b}\sqrt{b^2 - y^2}$

$$\bar{x} = \frac{\int x\, dA}{\int dA}$$

$$\int dA = \int_0^a y\, dx = \int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx = \frac{b}{a}\frac{\pi a^2}{4} = \frac{\pi ab}{4}$$

$$\int x\, dA = \int_0^a xy\, dx = \int_0^a x \frac{b}{a}\sqrt{a^2 - x^2}\, dx = \frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\, x\, dx$$



Let $a^2 - x^2 = t^2$

Differentiating both sides $\quad -2x\, dx = 2t\, dt$

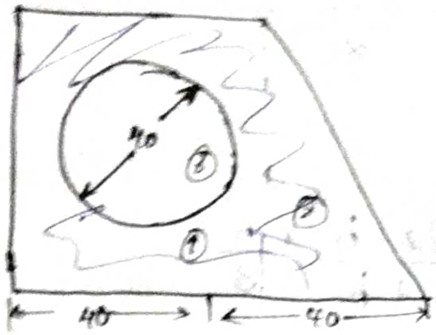
$x\, dx = -t\, dt$

$$\int x\, dA = \frac{b}{a}\int_0^a t[-t\, dt] = -\frac{b}{a}\left[\frac{t^3}{3}\right]_0^a$$

$$= -\frac{b}{3a}\left[(a^2 - x^2)^{3/2}\right]_0^a = \frac{b}{3a}\left[(a^2 - a^2)^{3/2} - (a^2 - 0)^{3/2}\right] = \frac{b}{3a}\left[0 - a^3\right] = \frac{ba^3}{3}$$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\frac{ba^3}{3}}{\frac{\pi ab}{4}} \rightarrow \bar{x} = \frac{4a}{3\pi} \qquad \bar{y} = \frac{4b}{3\pi}$$

(12)



$a_1 = 40 \times 60 = 2400\,mm^2$

$a_2 = \frac{1}{2} \times 40 \times 60 = 1200\,mm^2$

$a_3 = \pi r^2 = \pi\, 20^2 = 1256.64\,mm^2$

$y_1 = 60/2 = 30 \qquad y_2 = \frac{1}{3} \times 60 = 20\,mm$

$x_1 = 40/2 = 20\,mm$

$x_2 = 40 + \frac{1}{3} \times 40 = 53.33\,mm \qquad y_3 = \bar{y}$

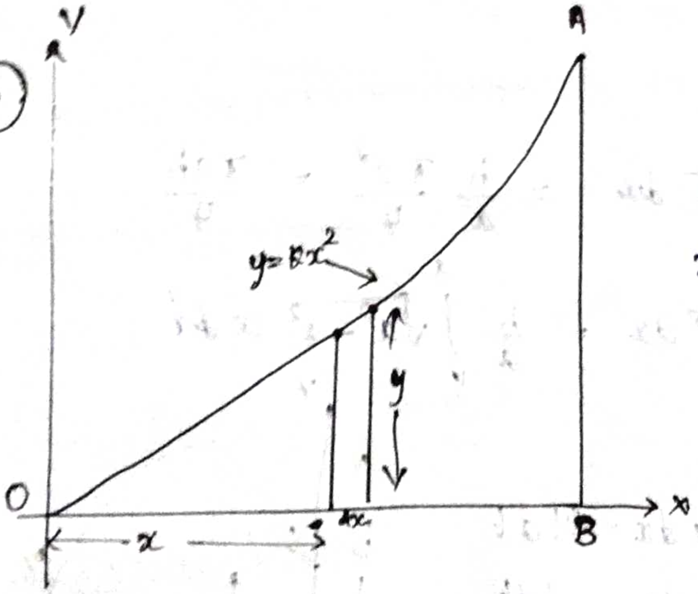
$$\bar{x} = \frac{2400 \times 20 + 1200 \times 53.33 - 1256.64\,\bar{x}}{2400 + 1200 - 1256.64}$$

$$2343.36\,\bar{x} + 1256.64\,\bar{x} = 48000 + 53.33 \times 1200$$

$$\bar{x} = 31.11\,mm$$

$$\bar{y} = \frac{2400 \times 30 + 1200 \times 20 - 1256.64\,\bar{y}}{2400 + 1200 - 1256.64} \Rightarrow \bar{y} = 26.67\,mm$$

(13)



$d_A = y\,dx = kx^2\,dx$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int_0^{25} x\,kx^2\,dx}{\int_0^{25} kx^2\,dx} = \frac{k\left[x^4/4\right]_0^{25}}{k\left[x^3/3\right]_0^{25}} = \frac{25^4/4}{25^3/3}$$

$$= \frac{3}{4} \times 25 = 18.75\,m$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\int_0^{25} \frac{kx^2}{2}\,kx^2\,dx}{\int_0^{25} kx^2\,dx} = \frac{\frac{k^2}{2}\left[x^5/5\right]_0^{25}}{k\left[x^3/3\right]_0^{25}} = \frac{3}{10}\,k\,25^2 = 187.5k$$

At A, $x = 25$, $y = 15$

$y = kx^2$

$15 = k \times 25^2$

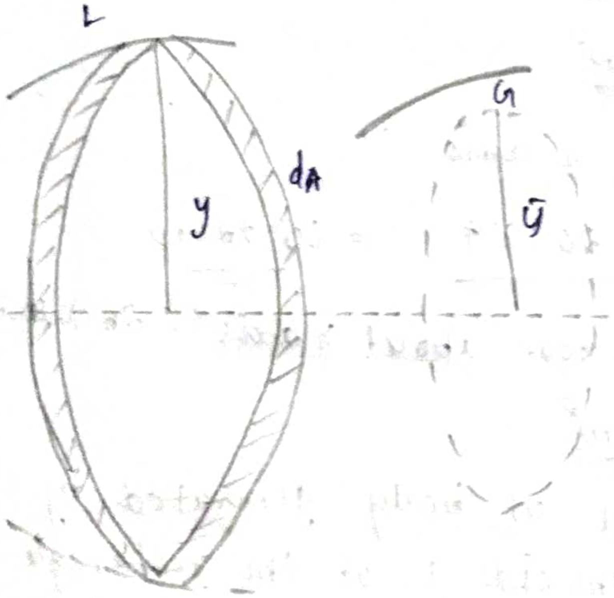
$\therefore k = \frac{15}{25^2}$

$\therefore \bar{y} = \frac{187.5 \times 15}{25^2} = 4.5 \text{ cm}$

THEOREM OF PAPPUS - GULDINUS

Theorem :- 01

The area of surface generated by revolving a plane curve about a non intersecting axes in the plane of curve = prdt of length of curve & distance travelled by the centroid of the curve while surface is being generated.



Consider an element of length dL.

The area generated by the element is equal to $2\pi y \, dL$.

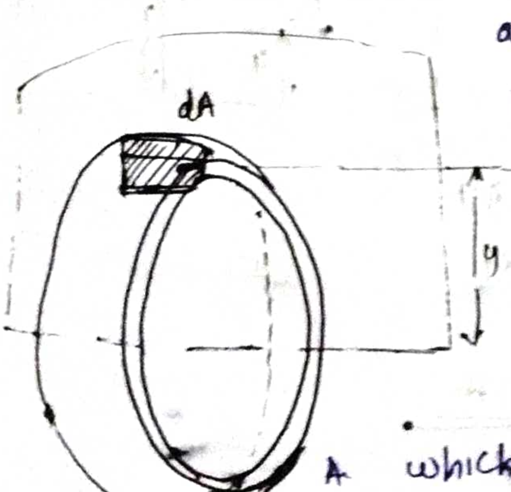
$$A = \int 2\pi y \, dL = 2\pi \int y \, dL$$

$$= 2\pi \bar{y} L$$

$2\pi \bar{y}$ is the distance travelled by the centroid of curve of length L.

Theorem :- 02

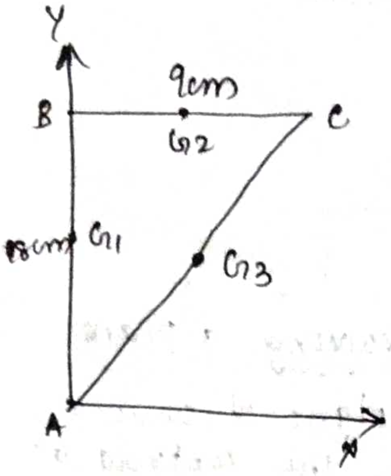
The volume of a body generated by revolving a plane area about a non-intersecting axis in the plane of area equal to the prdt of area = prdt of area & distance travelled by centroid of the plane area while the body is being generated.



Consider an element dA of the area which is revolved about x axis. The volume dv generated by element dA in one revolution = $2\pi y \, dA$

$$V = \int 2\pi y \, dA = 2\pi \int y \, dA = \underbrace{2\pi \bar{y}}_{\text{distance travelled by centroid of area A.}} A$$

(14) Calculate the S.A obtained by revolving the line ABC as shown in the figure about (i) x axis (ii) Y-axis



Length of line $= 18 + 9 + \sqrt{18^2 + 9^2}$

$= 18 + 9 + 20.1 = 47.1cm$

Distance of centroid from Y axis,

$$\bar{x} = \frac{L_1 x_1 + L_2 x_2 + L_3 x_3}{L_1 + L_2 + L_3}$$

$$= \frac{18 \times 0 + 9 \times 4.5 + 20.1 \times 4.5}{47.1} = \underline{2.78cm}$$

Distance travelled by centroid in one revolution about Y axis

$= 2 \times \pi \times 2.78cm$

Area $= 47.1 \times 2\pi \times 2.78 = \underline{822.7 cm^2}$

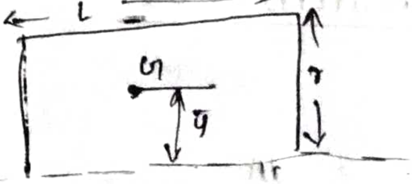
Distance of centroid from x axis

$$\bar{y} = \frac{9 \times 18 + 18 \times 9 + 20.1 \times 9}{47.1} = \underline{10.72cm}$$

Dist. travelled by centroid in one rev. about x axis $= 2\pi \times 10.72cm$

Area $= 47.1 \times 2\pi \times 10.72 = \underline{3172.46cm^2}$

(15) Obtain an expression for the vol of body generated by revol. of an rectangular area. The side L of the rectangle is in tough with axis of rotation & the other side is of length r.



Area of rectangle $= L \times r$

Distance of centroid from axis, $\bar{y} = r/2$

Distance travelled by centroid in one revolution $= 2\pi\bar{y} = 2\pi \frac{r}{2} = \pi r$

Volume of body generated $= L r \pi r = \pi r^2 L$

MOMENT OF INERTIA

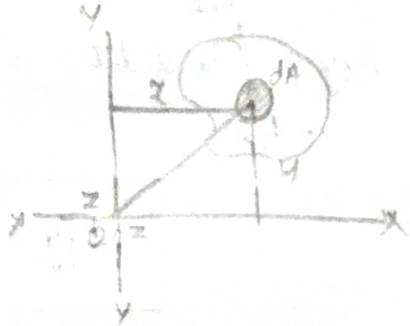
$$I_{AB} = \Sigma r^2 dA = \int r^2 dA$$

Radius of gyration $k = \sqrt{\frac{I}{A}}$

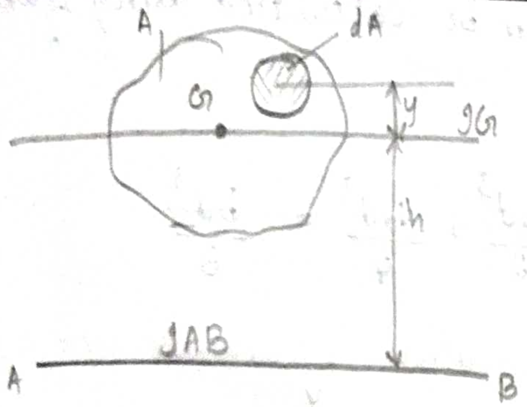
PERPENDICULAR AXIS THEOREM

If I_{xx} and I_{yy} are the moment of inertia of an area A about mutually $\perp^r$ axes XX & YY, in the plane of area, then the moment of inertia of the area about the z axis which is $\perp^r$ to XX & YY axis & passing through the point of intersection of XX & YY axis is given by,

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$



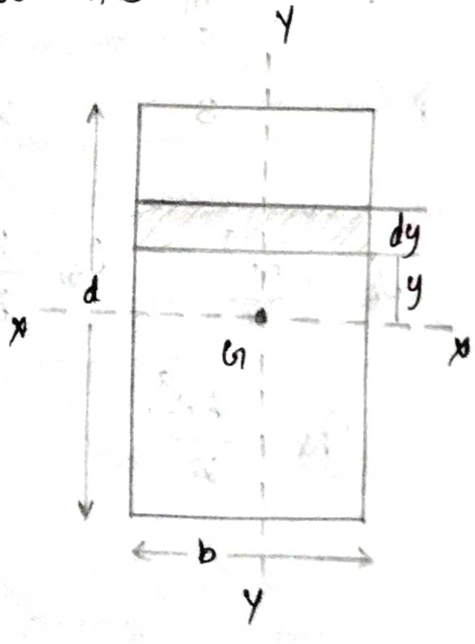
PARALLEL AXIS THEOREM



If I_G is the moment of inertia of a plane lamina of area A, about its centroidal axis in the plane of lamina, then the moment of inertia about any axis AB which is // to the centroidal axis and at a distance h from centroidal axis is given by,

$$\boxed{I_{AB} = I_G + Ah^2}$$

(16) Calculate the moment of inertia of a rectangular cross section about the centroidal axes and about its base AB



Area of element, $dA = b.dy$

Second moment of this elemental area about

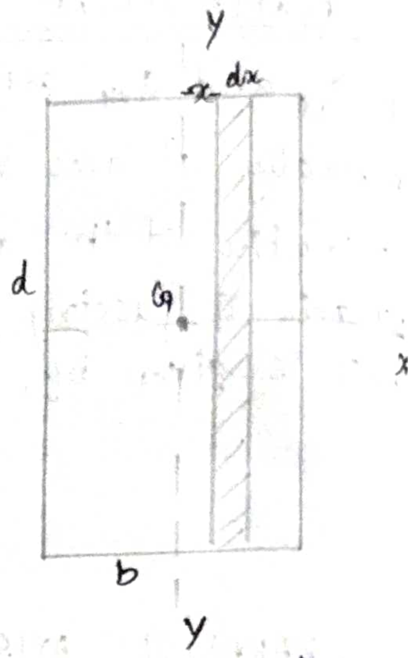
X axis =

$$dI_{xx} = y^2 dA = y^2 b.dy$$

$$I_{xx} = \int_{-d/2}^{d/2} y^2 b\, dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2} = \frac{b}{3}\left[\frac{d^3}{8} + \frac{d^3}{8}\right]$$

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

Area of element, $dA = d * dx$



Second moment of elemental area about Y axis

$$dI_{yy} = x^2 dA = x^2 d \cdot dx$$

$$I_{yy} = \int_{-b/2}^{b/2} x^2 d\, dx = d\left[\frac{x^3}{3}\right]_{-b/2}^{b/2} = \frac{d}{3}\left[\frac{b^3}{8}+\frac{b^3}{8}\right]$$

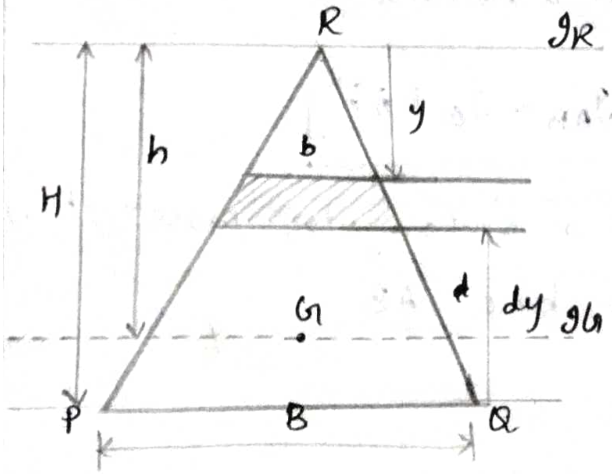
$$\boxed{I_{yy} = \frac{db^3}{12}}$$

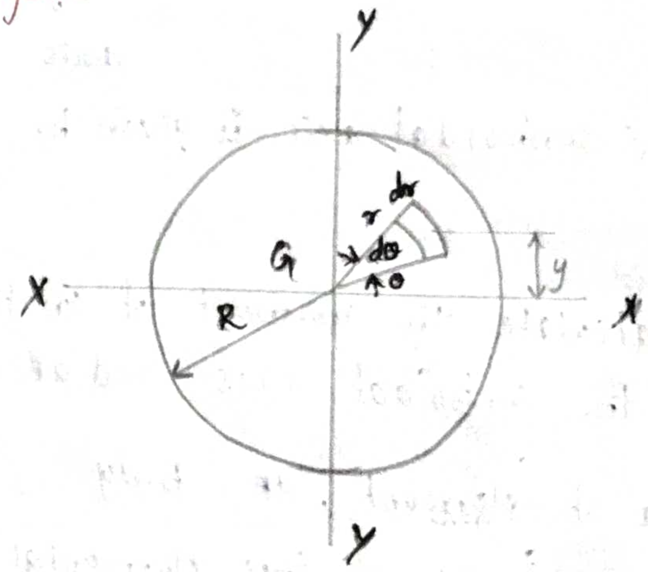
Moment of inertia about the base can be calculated using // axis

$$I_{AB} = I_G + Ah^2$$

$$= \frac{bd^3}{12} + bd \times \left(\frac{d}{2}\right)^2 = \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{3}$$

Moment of inertia of a rectangular lamina about its base is $\frac{bd^3}{3}$



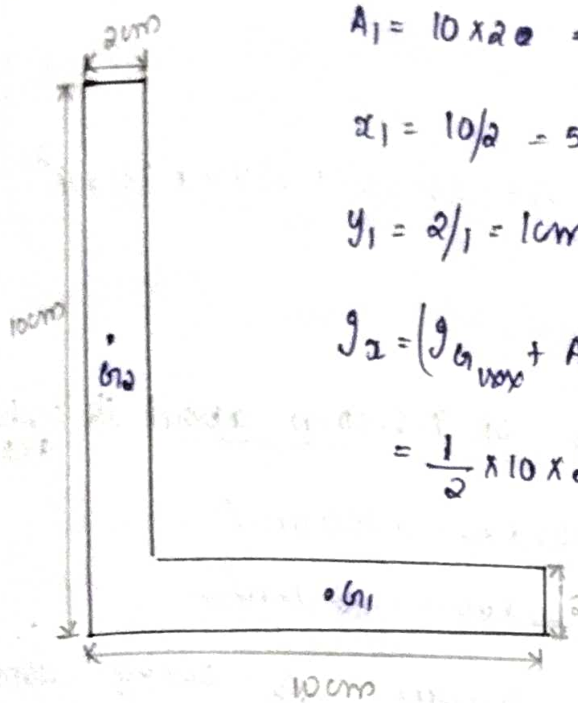


$$I_R = \frac{BH^3}{4} \qquad I_G = \frac{BH^3}{36}$$

$$I_{yy} = I_{xx} = \frac{\pi R^4}{4}$$

$$I_{PQ} = \frac{BH^3}{12}$$

(17) Calculate the M.I of angle section having the dimensions as



$A_1 = 10 \times 2 = 20 cm^2$ $A_2 = 8 \times 2 = 16 cm^2$

$x_1 = 10/2 = 5cm$ $x_2 = 2/2 = 1 cm$

$y_1 = 2/1 = 1cm$ $y_2 = 2 + 8/2 = 6cm$

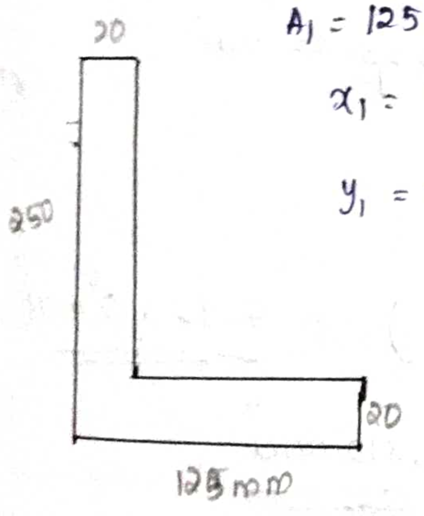
$I_x = (I_{G_1xx} + A_1 y_1^2) + (I_{G_2xx} + A_2 y_2^2)$

$= \frac{1}{2} \times 10 \times 2^3 + 20 \times 1^2 + \frac{1}{12} \times 2 \times 8^3 + 16 \times 6^2 = 688 cm^4$

$I_y = I_{G_1yy} + A_1 X_1^2 + I_{G_2yy} + A_2 X_2^2$

$= \frac{1}{12} \times 2 \times 10^3 + 20 \times 5^2 + \frac{1}{12} \times 8 \times 2^3 + 16 \times 1^3 = 688 cm^4$

(18) Calculate the M.I of an unequal angle iron section of 250 mm × 20mm about its centroid axes



$A_1 = 125 \times 20 = 2500 mm^2$ $A_2 = 230 \times 20 = 4600 mm^2$

$x_1 = 125/2 = 62.5mm$ $x_2 = 20/2 = 10mm$

$y_1 = 20/2 = 10mm$ $y_2 = 20 + \frac{230}{2} = 135mm$

$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = 28.49 mm$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = 91 mm.$

$I_{G_1xx} = \frac{1}{12} \times 125 \times 20^3 = 83333.33 mm^4$ $I_{G_2xx} = \frac{1}{12} \times 120 \times 230^2 = 20278333.33 mm^4$

$I_{G_1yy} = \frac{1}{12} \times 20 \times 125^3 = 3255208.33 mm^4$ $I_{G_2yy} = \frac{1}{12} \times 230 \times 20^3 = 153333.33 mm^4$

$h_1 = GG_1 = \bar{y} - y_1 = 91 - 10 = 81mm$ $h_2 = GG_2 = y_2 - \bar{y} = 135 - 91 = 44mm$

$I_{G_{xx}} = I_{G_1xx} + A_1 h_1^2 + I_{G_2xx} + A_2 h_2^2$

$= 83333.33 + (2500 \times 81^2) + 20278333.33 + 4600 \times 44^2$

$= 45669766.66 mm^4$

h_1 is the horizontal distance GG_1 & h_2 is the horizontal distance GG_2

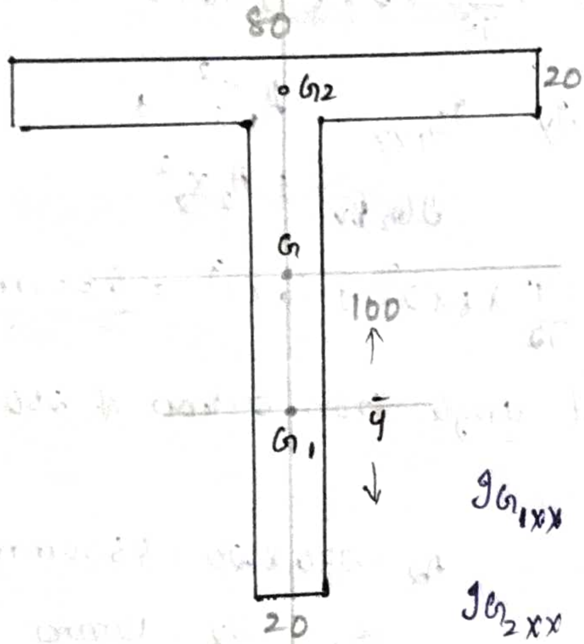
$h_1 = x_1 - \bar{x} = 62.5 - 28.49 = 34.01 mm$

$h_2 = \bar{x} - x_2 = 28.49 - 10 = 18.49 mm$

$I_{G_{yy}} = 8255208.33 + 2500 \times 34.01^2 + 153333.33 + 4600 \times 18.49^2$

$= 7872887.37 mm^4$

(19) Determine the moment of inertia of T-section about its centroid axis



$A_1 = 100 \times 20 = 2000 mm^2$

$A_2 = 80 \times 20 = 1600 mm^2$

$y_1 = \frac{100}{2} = 50mm$ $\quad y_2 = 100 + \frac{20}{2} = 110mm$

$\bar{y} = \frac{2000 \times 50 + 1600 \times 110}{2000 + 1600} = 76.67$

$I_{G_{1xx}} = \frac{20 \times 100^3}{12} = 1666666.67 mm^4$

$I_{G_{2xx}} = \frac{80 \times 20^3}{12} = 53333.33 mm^4$

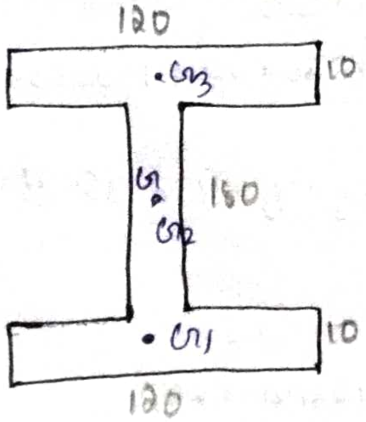
$h_1 = \bar{y} - y = 76.67 - 50 = 26.67$

$h_2 = y_2 - \bar{y} = 110 - 76.76 = 33.33$

$I_{Gxx} = (I_{G_{1xx}} + A_1 h_1^2) + (I_{G_{2xx}} + A_2 h_2^2) = 4.92 \times 10^6 mm^4$

$I_{Gyy} = \frac{100 \times 20^3}{12} + \frac{20 \times 80^3}{12} = 9.2 \times 10^5 mm^4$

(20)



$I_{GG} = I_{G_1yy} + I_{G_2yy} + I_{G_3yy}$

$= \frac{1}{12}\left[10 \times 120^3 + 180 \times 10^3 + 10 \times 120^3\right]$

$= 2895000 mm^4$

$I_{G_{1xx}} = I_{G_{3xx}} = \frac{1}{12} \times 120 \times 10^3 = 10^4 mm^4$

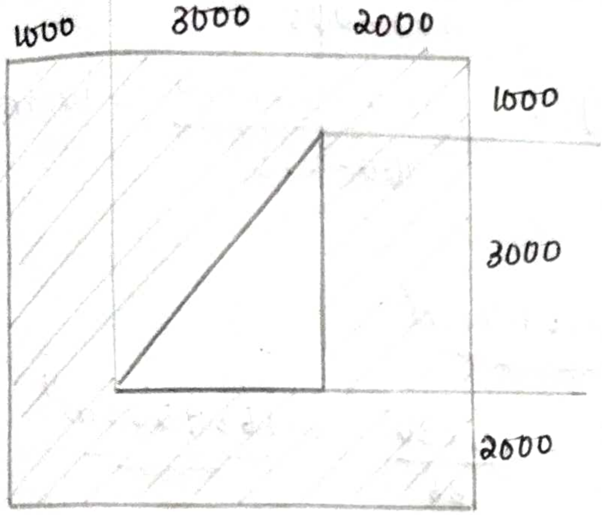
$$I_{G_2xx} = \frac{1}{12} \times 10 \times 180^3 = 4860000 \text{ mm}^4$$

$$\cancel{I_{G_3xx} = \frac{1}{12} \times 10 \times 180^3} \qquad h_1 = h_3 = 100 - 5 = 95 \text{ mm}, \quad h_2 = 0$$

$$I_{Gxx} = (10000 + 1200 \times 95^2) \times 2 + (4860000 + 180 \times 10 \times 0)$$

$$= \underline{26540000 \text{ mm}^4}$$

(21) M.I of shaded area w.r.t the centrodial axes.



$$a_1 = 6 \times 10^3 \times 6 \times 10^3 = 36 \times 10^6 \text{ mm}^2$$

$$a_2 = \frac{1}{2} \times 3 \times 3 \times 10^6 = 4.5 \times 10^6 \text{ mm}^2$$

$$x_1 = 3 \times 10^3 \text{ mm} \qquad y_1 = 3 \times 10^3 \text{ mm}$$

$$x_2 = \frac{2}{3} \times 3000 + 1000 = 3 \times 10^3 \text{ mm}$$

$$y_2 = \frac{1}{3} \times 3000 + 2000 = 3 \times 10^3 \text{ mm}$$

$$\bar{x} = \frac{36 \times 10^6 \times 3 \times 10^3 - 4.5 \times 10^6 \times 3 \times 10^3}{36 \times 10^6 - 4.5 \times 10^6} = 3 \times 10^3 \text{ mm}$$

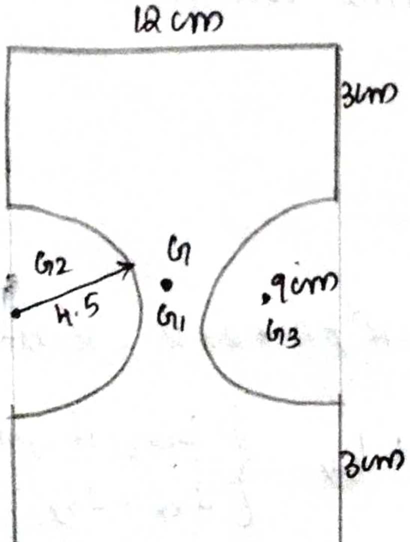
$$I_{Gxx} = I_{G_1xx} - I_{G_2xx} = \frac{1}{12} \times 6 \times 10^3 \times (6 \times 10^3)^3 - \frac{3 \times 10^3 \times (3 \times 10^3)^3}{36}$$

$$= \underline{105.75 \times 10^{12} \text{ mm}^4}$$

$$I_{Gyy} = I_{G_1yy} - I_{G_2yy} = \frac{1}{12} \times 6 \times 10^3 \times (6 \times 10^3)^3 - \frac{3 \times 10^3 \times (3 \times 10^3)^3}{36}$$

$$= \underline{105.75 \times 10^{12} \text{ mm}^4}$$

(22)



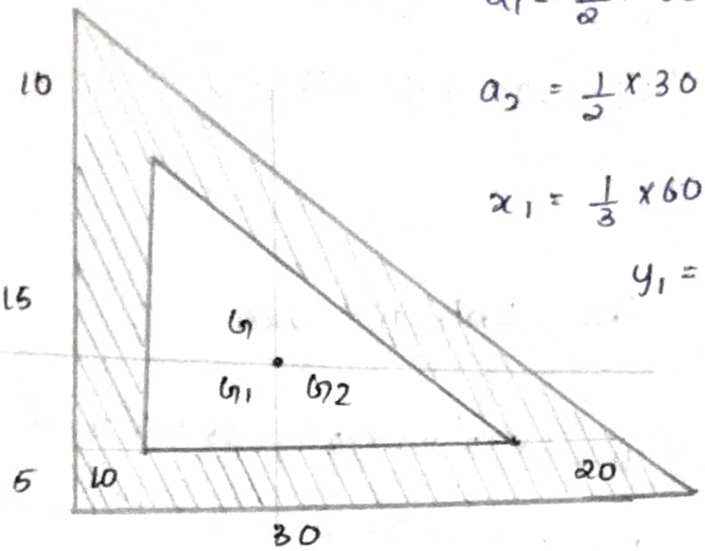
$$I_{Gxx} = I_{G_1xx} - I_{G_2xx} - I_{G_3xx}$$

$$= I_{G_1xx} - 2 \times I_{G_2xx}$$

$$I_{Gyy} = I_{G_1yy} - (I_{G_2yy} + A_2 h_2^2) - (I_{G_3yy} + A_3 h_3^2)$$

$$= I_{G_1yy} - 2(I_{G_2yy} + A_2 h_2^2)$$

(23) Determine the M.I of shaded area w.r.t centroidal axes



$a_1 = \frac{1}{2} \times 60 \times 30 = 900 cm^2$

$a_2 = \frac{1}{2} \times 30 \times 15 = 225 cm^2$

$x_1 = \frac{1}{3} \times 60 = 20$ $\quad x_2 = \frac{1}{3} \times 30 + 10 = 20$

$y_1 = \frac{1}{3} \times 30 = 10$ $\quad y_2 = \frac{1}{3} \times 15 + 5 = 10$

$$\bar{x} = \frac{900 \times 20 - 225 \times 20}{900 - 225} = 20 cm$$

$$\bar{y} = \frac{900 \times 10 - 225 \times 10}{900 - 225} = 10 cm$$

$$I_{Gxx} = I_{G_1xx} - I_{G_2xx}$$

$$= \frac{60 \times 30^3}{36} - \frac{30 \times 15^3}{36} = \underline{42187.5 cm^4}$$

$$I_{Gyy} = I_{G_1yy} - I_{G_2yy} = \frac{30 \times 60^3}{36} - \frac{15 \times 30^3}{36} = \underline{168750 cm^4}$$

Note

Translatory inertia is defined as mass & rotational inertia is known as moment of inertia.

MASS MOMENT OF INERTIA

unit - kgm^2

The moment of inertia of a body about an axis at a distance d and // to centroidal axis is = sum of M.I about centroidal axis and produce of mass & sq. of distance b/w parallel axes

$$\boxed{I = I_G + md^2}$$

Mass moment of inertia of ring

M.I of ring about zz axis

$$I_{zz} = \int_0^{2\pi R} \rho A \, dl \times R^2 = R^2 \rho A [l]_0^{2\pi R} = R^2 \rho A \times 2\pi R = R^2 2\pi R A \rho$$

$$\boxed{I_{zz} = mR^2}$$

$I_{zz} = I_{xx} + I_{yy}$ $\quad \left\{ \begin{array}{l} \text{bcoz of sym.} \\ I_{xx} = I_{yy} \end{array} \right.$

$$I_{zz} = I_{xx} + I_{yy} = 2I_{xx} \qquad \boxed{I_{xx} = I_{yy} = \frac{MR^2}{2}}$$

Mass moment of inertia of a disc

$$I_{zz} = \int_0^R 2\pi r\, dr\, t\rho r^2 = 2\pi t\rho \int_0^R r^3 dr = 2\pi t\rho \left[\frac{r^4}{4}\right]_0^R = (\pi R^2 t\rho)\frac{R^2}{2} = \frac{mR^2}{2}$$

polar moment of inertia, $I_{zz} = I_{xx} + I_{yy} = \frac{mR^2}{2}$

$$\boxed{I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{mR^2}{4}}$$

Mass moment of inertia of a cylinder

Mass of element, $dm = \pi R^2 dy\, \rho$

Moment of inertia, thin circular disc about its centroidal xx axis

$$dI = \frac{dm R^2}{4} \Rightarrow dI_{xx} = dI + (dm)y^2$$

$$dI_{xx} = \left[\frac{dm R^2}{4} + dm y^2\right] dm = \pi R^2 dy \frac{\rho R^2}{4} + \pi R^2 dy \times \rho y^2$$

$$I_{xx} = \int_{-h/2}^{h/2} \left(\frac{\pi R^2}{4}\rho\right) dy + \pi R^2 \rho \int_{-h/2}^{h/2} y^2 dy = 2\pi \frac{R^4}{4}\rho\,[y]_0^{h/2} + 2\pi R^2 \rho \left[\frac{y^3}{3}\right]_0^{h/2}$$

$$= \frac{M}{4}\left(\frac{3R^2 + h^2}{3}\right) = \frac{M}{12}(3R^2 + h^2)$$

$$\boxed{I_{zz} = I_{xx} = \frac{M}{12}(3R^2 + h^2)}$$