MODULE : 2.

Friction & Parallel Coplanar Forces.

Friction or Frictional force.

Whenever a body moves or tends to move over other body; a force opposite to the direction in which the body moves or tends to move is developed at the contact surface.

cause : Surface roughness.

perfectly smooth surface, = $F_r = 0$

Friction → static friction, kinetic friction

static friction

- No relative motion

$\boxed{F_S = \mu_s F_n}$

kinetic friction

- Relative motion
- Independent of velocity

$\boxed{F_k = \mu_k F_n}$

As force increases, friction increases.

Max value of friction : Limiting friction.

↓

Value of frictional force when motion is about to start.

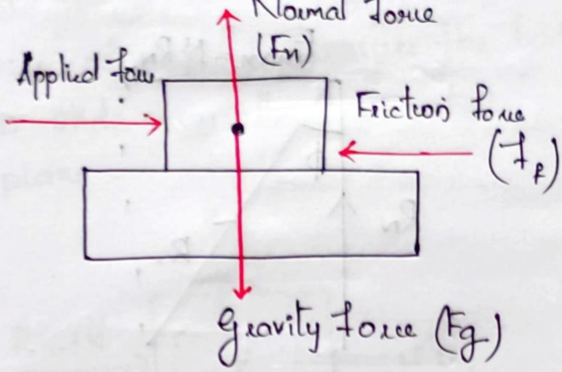
* Applied force >>> Limiting friction

→ Body moves in the direction of applied force

* Applied force <<< Limiting friction

→ Body remains at rest.

Free body diagram.

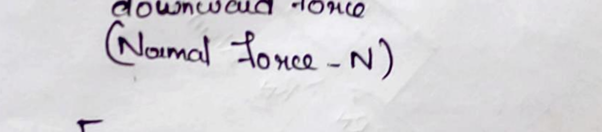


$F_g = mg$

$F_N = F_g$ (No motion)

$F = f_f$.

Coefficient of friction (μ)



$F_{friction} = \mu N$

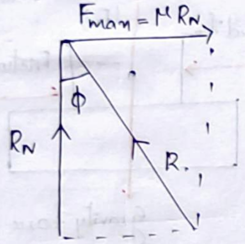
$F_{max} \propto R_N$

$= \mu R_N$

$\mu = \dfrac{F_{max}}{R_N}.$

Angle of friction (ϕ)

→ Angle b/w contact surface & normal reaction

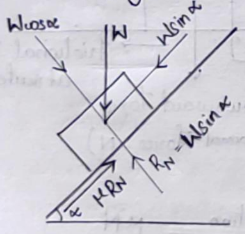


$$\tan\phi = \frac{\mu R_N}{R_N} = \mu.$$

Angle of friction $\phi = \tan^{-1}\mu$

Angle of repose (α)

Maximum inclination of a plane on which a body remains at rest, without applying external force.



$F = \mu \times R_N.$

$\Sigma F_n = 0$

$\mu R_n - W\sin\alpha = 0$

$\mu R_N = W\sin\alpha.$

$\Sigma F_y = 0$

$R_N - W\cos\alpha = 0$

$R_N = W\cos\alpha$

$\mu W\cos\alpha = W\sin\alpha$

$\tan\alpha = \mu = \tan\phi$

$\therefore \alpha = \phi$

Angle of repose = Angle of friction.

Coulomb's Laws of friction

1. The maximum friction that can be developed is independent of the area of contact.
2. At low velocity, the frictional force is independent of velocity of the contact surface.
3. The maximum frictional force is proportional to the normal reaction at the contact surface.

Other laws of friction

1. The force of friction always acts in a direction opposite to the direction in which the body moves or tends to move.
2. Till the limiting value is reached, the magnitude of friction is equal to the external force which tends to move the body.
3. The force of friction depends on the roughness of the surface in contact. When two perfectly smooth surfaces are in contact, frictional force is zero.

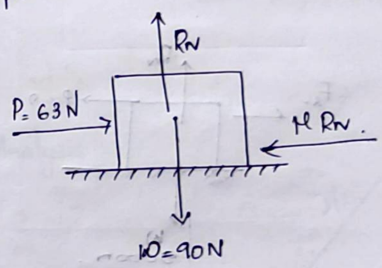
Analysis of single bodies :—

ⓐ consider single body

ⓑ various forces resolve horizontally & vertically

ⓒ apply conditions of equilibrium.

Q: A body of weight 90N is placed on a rough horizontal plane. Determine the coefficient of friction if the horizontal force of 63N just causes the body to slide over the horizontal plane.



$\Sigma F_y = 0$

$R_N - W = 0$

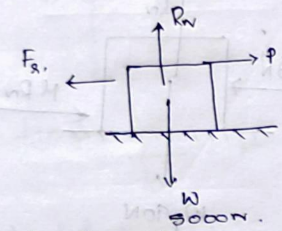
$R_N = W = 90N$

$\mu R_N = P = 63N$

$\mu R_N = 63N$

$$\mu = \frac{63}{R_N} = \frac{63}{90} = 0.7$$

Q. A uniform wooden cube of side 1m and weighing 500N rests on its side on a horizontal plane. Find the max horizontal force can be applied at the top edge of the cube to just make it slide without overturning. The coefficient of friction is 0.25



$\Sigma M = 0$ (overturning)

$F_R = .25 \times R_N$

$\Sigma F_v = 0 \quad R_N - W = 0$

$R_N = W = 5000N$

$\Sigma F_h = 0 \quad P - \mu R_N = 0$

$P = \mu R_N = 0.25 \times 5000$

$= \underline{1250N}$

$\Sigma M = 0$ abt A.

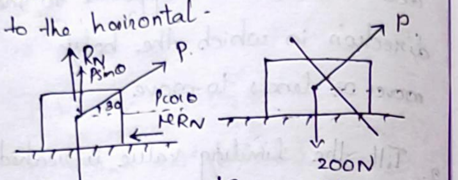
$P \times 1 - W \times 0.5 = 0.$

$P = W \times 0.5$

$= 5000 \times .5$

$= \underline{2500N}$

Q. A block weight 200N is placed on a rough horizontal floor. If $\mu = .25$ find the pull P required to move the block if P is inclined upwards at 30° to the horizontal.



$R_N + P\sin\theta - 200 = 0$

$R_N = 200 - P\sin\theta$

$P\cos\theta - \mu R_N = 0$

$P\cos\theta = \mu R_N$

$= 0.25(200 - P\sin 30)$

$P = \underline{50.45}$

Wedge friction

Analysis:—

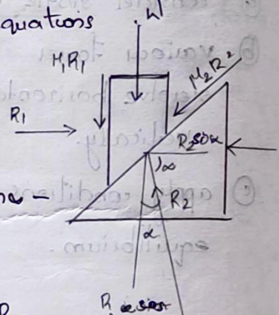
(a) fig

(b) main forces (W & P_{ext}), wedge angle

(c) consider body

(d) consider wedge

(e) solve equations

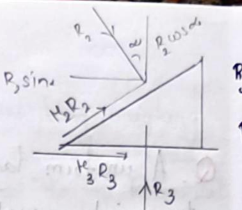


$\Sigma F_v = 0$

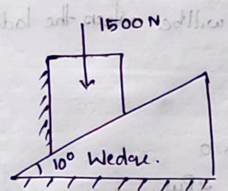
$R_2\cos\alpha - \mu_2 R_2 \sin\alpha - \mu_1 R_1 - W = 0$

$\Sigma F_H = 0$

$R_1 - \mu_2 R_2 \cos\alpha - R_2 \sin\alpha = 0$

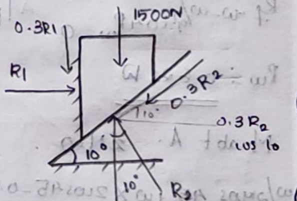


Q. Find the horizontal force P on the 10° wedge shown in the fig to raise the 1500 N load. The coefficient of friction is 0.3 at all contact surfaces.



Soln:

consider equilibrium of Load,



$R_1 - 0.3R_2\cos 10 - R_2 \sin 10 = 0.$

$R_1 = R_2(0.3\cos 10 + \sin 10)$

$= 0.47 R_2 \longrightarrow (1)$

$R_2\cos 10 - 0.3R_2 \sin 10 - 1500 - 0.3R_1 = 0$

$R_2\cos 10 - 0.3 R_2 \sin 10 - 0.3(0.47 R_2) = 1500$

$R_2(\cos 10 - 0.3\sin 10 - 0.3 \times 0.47) = 1500$

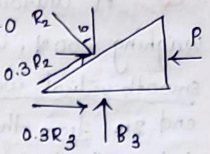
$R_2 = 1894.63N$

consider equilibrium of wedge,

$R_3 + 0.3R_2 \sin 10 - R_2\cos 10 = 0$

$R_3 = R_2(\cos 10 - 0.3\sin 10)$

$= \underline{1767.15N}$

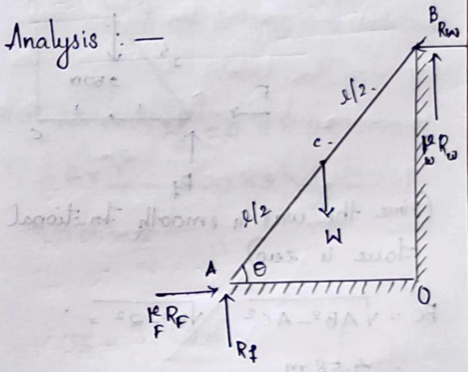


$0.3R_2\cos 10 + R_2 \sin 10 + 0.3R_3 - P = 0$

$P = \underline{1418.90N}$

Ladder friction

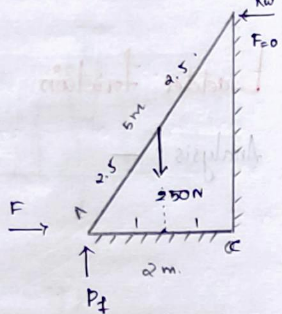
Analysis:—



(i) upper end B slips down → friction is upwards.

(ii) lower end moves away from the wall → direction of friction force is towards the wall.

Q. A uniform ladder 5m long weighing 250N, is placed against a smooth vertical wall with its lower end 2m from the wall. The coefficient between the ladder and floor is 0.25. show that ladder will remain in equilibrium in this position.



(Since the wall is smooth frictional force is zero)

$BC = \sqrt{AB^2 - AC^2} = \sqrt{5^2 - 2^2} = 4.58\ m$

$\Sigma F_V = 0$

$R_f - 250 = 0$

$R_f = 250$

Limiting frictional force $= 0.25 \times 250 = 62.5\ N$

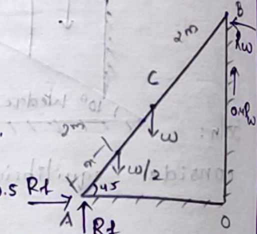
Moment abt B,

$\Sigma H = 0$

$R_f \times AC - 250 \times 1 - F \times BC = 0$

$F = 54.59 N$

Q. A uniform ladder of 4m length rests against a wall which it makes an angle 45° as shown in fig. The coefficient of friction b/w the ladder and the wall is 0.4 & that between the ladder and floor is 0.5. If a man whose weight is half of that of a ladder ascends it, how high will be when the ladder slips?



Soln:

$\Sigma F_H = 0$

$0.5R_f - R_w = 0$

$R_f = 2R_w$

$\Sigma F_V = 0$

$R_f - w - w/2 + 0.4R_w = 0$

$R_w = 0.625\ W$

M abt A, $\Sigma H = 0$

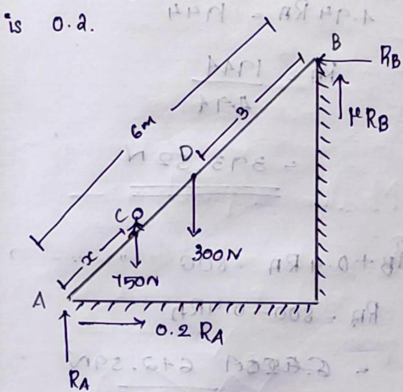
$w/2\, x \cos 45 + w \times 2\cos 45 - 0.4R_w \times 4\cos 45 - R_w\, 4\sin 45 = 0$

$x/2 = 1.5$

$x = 3\ m$

Tutorial

Q1. The ladder 6m long weighing 300N is resting against a wall at an angle of 60° to the horizontal. A man weighing 750N climbs the ladder. At what position along the ladder from the bottom does the induce slipping? The coefficient of friction for both the wall and the ground with the ladder is 0.2.



$\Sigma F_x = 0$

$0.2R_A - R_B = 0$

$0.2R_A = R_B$

$R_A = 5R_B$

$\Sigma F_y = 0$

$R_A + 0.2R_B - 300 - 750 = 0$

$5R_B + 0.2R_B = 1050$

$5.2 R_B = 1050$

$R_B = \frac{1050}{5.2} = 201.9\ N$

$R_A = 5R_B = 5 \times 201.9 N = 1009.62 N$

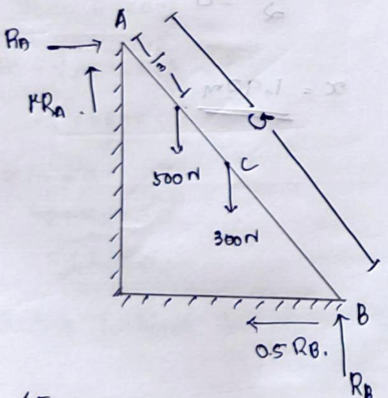
Moment about A

$R_B . 6\sin 60 + 0.2R_B \times 6\cos 60 - 300 \times 3\cos 60 - 750 \times x\cos 60 = 0$

$201.92 \times 6 \times \frac{\sqrt{3}}{2} - 0.2 \times 201.92 \times 6 \times \frac{1}{2} - 300 \times 3 \times \frac{1}{2} - 750 \times \frac{x}{2} = 0$

$x = 1.92m$

Q.2. A ladder of length 5 m and weight 300N is placed against a vertical wall with which it makes an angle of 45°. The coefficient of friction between the floor and the ladder is 0.5 and that between the ladder & 0.4. In addition to its own weight, the ladder has to support a man of weight 500N at 1m from the top along the ladder. Determine the minimum horizontal force 'P' to be applied at the floor level to prevent the ladder from slipping.



$\Sigma F_y = 0$

$R_B + 0.4R_A - 300 - 500 = 0$

$R_B + 0.4 R_A = 800 \rightarrow (1)$

$\Sigma F_x = 0$

$R_A - 0.5R_B - P = 0$

$R_A - 0.5 R_B = P \rightarrow (2)$

Moments about B = 0

$\Sigma M = 0$

$R_A \cdot 5\cos 45 + 0.4R_A \cdot 5\cos 45 - 300 \times 2.5\cos 45 - 500 \times 4\cos 45 = 0$

$R_A(5\cos 45 + .4\cos 45) - 530 - 1414 = 0$

$4.94 R_A = 1414 + 530$

$4.94 R_A = 1944$

$R_A = \frac{1944}{4.94}$

$= 393.52\,N$

$R_B + 0.4 R_A = 800$

$R_B = 800 - 0.4R_A$

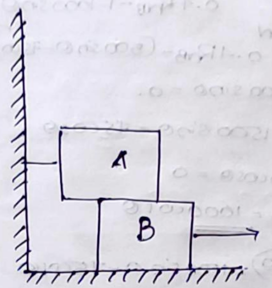
$= \cancel{6.4264}\ 642.59N$

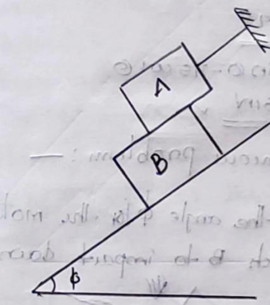
$P = R_A - 0.5 R_B$

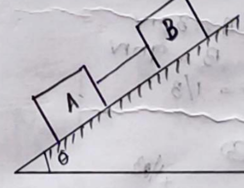
$= 393.52 - 0.5 \times 642.59$

$= 72.225N$

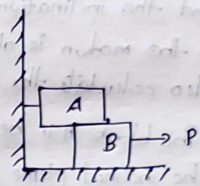
Analysis of friction in connected bodies.



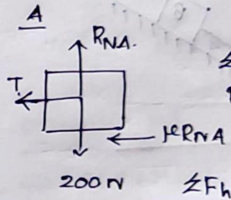




Q. Block A shown in figure weights 200N and block B weight 300N. Find the force P required to move block B. Assume the coefficient of friction for all surface as 0.3.



A



$\Sigma F_v = 0$

$R_{NA} - W = 0$

$R_{NA} = W = 200N$

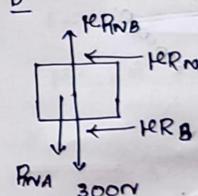
$\Sigma F_h = 0$

$\mu R_{NA} - T = 0$

$.3 \times 200 = T$

$T = 60N$

B



$\Sigma F_v = 0$

$R_{NB} - W - R_{NA} = 0$

$R_{NB} = W + R_{NA}$

$= 300 + 200 = 500N$

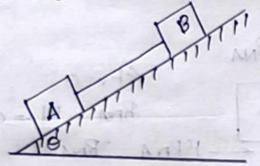
$\Sigma F_h = 0$

$P - \mu R_{NB} - \mu R_{NA} = 0$

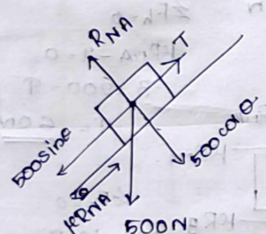
$P = \mu R_{NB} + \mu R_{NA}$

$.3 \times 500 + 0.3 \times 200 = 210N$

Q: Two blocks A & B of weights 500N & 1000N are placed on an inclined plane. The blocks are connected by a string parallel to the inclined plane. The coefficient of friction between the inclined plane and block A is 0.15 and that for block B is 0.4. Find the inclination of the plane when the motion is abt to take place. Also calculate the tension in the string. The block A is below Block B on the inclined plane



A



$T + 0.15\,R_{NA} - 500\sin\theta = 0$.

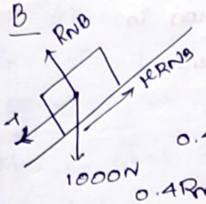
$R_{NA} - 500\cos\theta = 0$

$R_{NA} = 500\cos\theta$.

$T + 0.15 \times 500\cos\theta - 500\sin\theta = 0$.

$T = 500\sin\theta - 75\cos\theta$.

B



$0.4R_{NB} - T - 1000\sin\theta = 0$

$0.4R_{NB} - (500\sin\theta - 75\cos\theta)$

$- 1000\sin\theta = 0$.

$0.4R_{NB} = 1500\sin\theta - 75\cos\theta$

$R_{NB} - 1000\cos\theta = 0$

$R_{NB} = 1000\cos\theta$

$0.4(1000\cos\theta) = 1500\sin\theta - 75\cos\theta$

$400\cos\theta = 1500\sin\theta - 75\cos\theta$

$475\cos\theta = 1500\sin\theta$

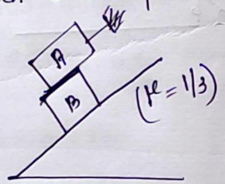
$\theta = 17.57^\circ$

$T = 500\sin\theta - 75\cos\theta$

$= 79.61N$ ✓

Miscellaneous problems :—

Q. Find the angle ϕ for the motion of block B to impart down the plane.

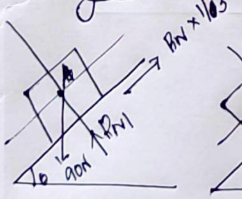


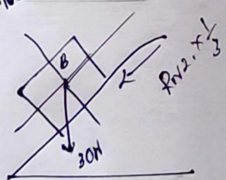
$A = 30N$

$B = 90N$

$\mu = 1/3$

$\sum F_y = 0$





$\sum F_y = 0$

$R_{N2} - 30\cos\theta = 0$

$R_{N2} = 30\cos 30 \rightarrow (1)$

$\sum F_x = 0$

$T - \frac{R_{N2}}{3} - 30\sin 30 = 0$

$\therefore T = \frac{R_{N2}}{3} + 30\sin\theta \rightarrow (2)$

$T = \frac{\overset{10}{\cancel{30}}\cos 30}{\cancel{3}} + 30\sin\theta \longrightarrow (2)$

$\sum F_y = 0$

$R_{N1} - R_{N2} - 90\cos\theta = 0$.

$\Rightarrow R_{N1} = 120\cos\theta \rightarrow (3)$

$\sum F_x = 0$

$\frac{R_{N1}}{3} + \frac{R_{N2}}{3} - 90\sin\theta = 0$

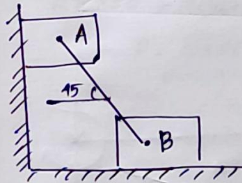
$R_{N1} + R_{N2} = 270\sin\theta$

$R_N = 270\sin\theta - 30\cos\theta \rightarrow (4)$

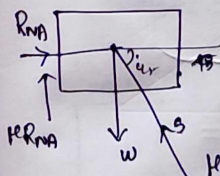
$\Rightarrow (4) \Rightarrow 120\cos\theta = 270\sin\theta - 30\cos\theta$

$\theta = 29.055^\circ$

Q. Two identical Blocks A & B of weight w are supported by a rigid bar inclined 45° with the horizontal as shown in fig. If the blocks are in limiting equilibrium, find the coefficient of friction, assuming it to be the same at the floor and the wall.



A



$\sum F_h = 0$

$R_{NA} - S\cos 45 = 0$

$R_{NA} = 0.707S$

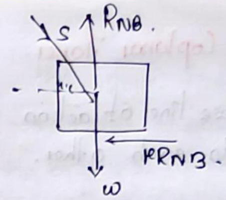
$\sum F_v = 0$

$\mu R_{NA} + S\sin 45 - W = 0$

$\mu \times .707S + 0.707S = W$

$0.707S(1 + \mu) = W \rightarrow (1)$

B



$\sum F_V = 0$

$R_{NB} - W - S\sin 45 = 0$

$R_{NB} = W + .707S$.

$\sum F_h = 0$

$S\cos 45 - \mu R_{NB} = 0$

$S\cos 45 - \mu(W + .707S) = 0$

$.707S - \mu \cdot 0.707S = \mu W$

$.707S(1 - \mu) = \mu W \rightarrow (2)$

$\frac{0.707S(1-\mu)}{.707S(1+\mu)} = \frac{\mu W}{W}$

$1 - \mu = \mu(1 + \mu)$

$1 - \mu = \mu + \mu^2$

$\mu^2 + 2\mu - 1 = 0$

$\mu = \frac{-2 + \sqrt{8}}{2} = \frac{-2 + 2\sqrt{2}}{2}$

$= 0.414$

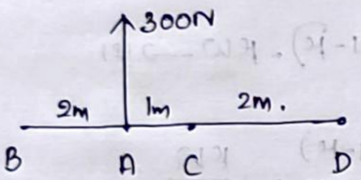
Parallel Coplanar forces.

Force whose line of action are parallel to each other.

Q1. Resolve the force of 300N as shown in fig. into two parallel components

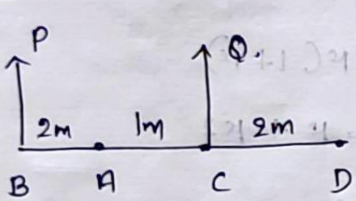
(I) at B & C

(II) at C & D.



soln:

(I)



Moment at B due to 300N

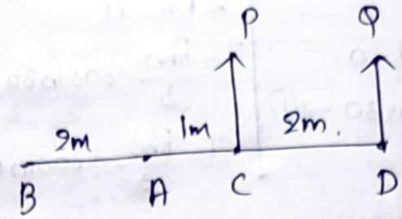
$= 300 \times 2 = 600\,Nm$

$P \times 0 + Q \times 3 = 600$

$Q = 200N$

$P + Q = 300$

$P = 100N$



$P+Q=300$

Moment at D due to 300N

$= 300 \times 3 = 900.$

$P \times 2 + Q \times 0 = 900$

$P = 450$

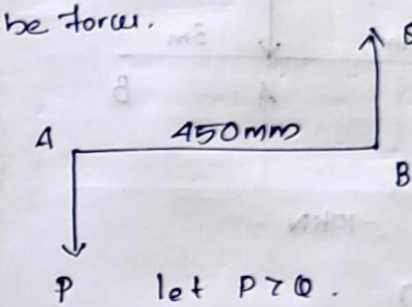
$P + Q = 300$

$450 + Q = 300$

$Q = 300 - 450$

$= -150N$

Q. Two unlike parallel forces are acting at a distance of 450 mm from each other. The forces are equivalent to a single force of 900N, which act a distance of 200mm from the greater of the two forces. Find the magnitude of the forces.



let $P > Q$.

$P - Q = 900 \rightarrow (1)$

Moment of B P abt B must be equal to moment of 900N abt B.

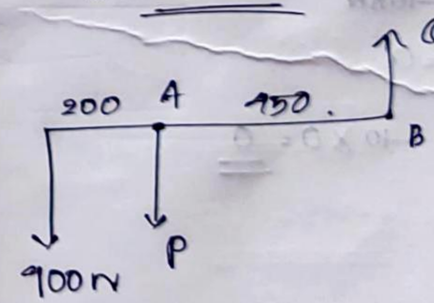
$P \times 450 = 900 \times 450$

$P = 1300N$

$1300 - Q = 900$

$1300 - 900 = Q$

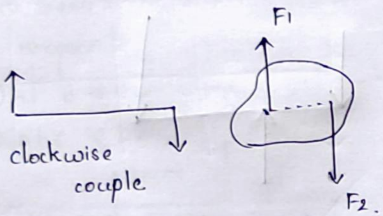
$Q = 400N$



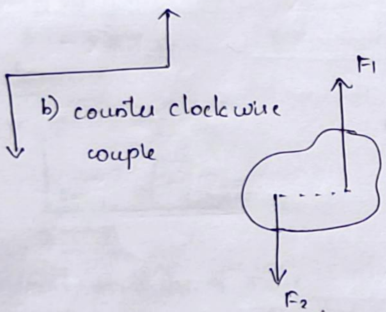
Couple.

Two forces having the same magnitude, parallel line of action and opposite in direction forms a couple.

Resultant of these forces is zero.



a) clockwise couple

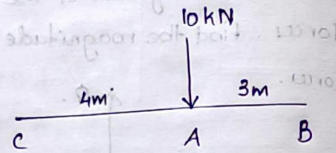


b) counter clockwise couple

Q. Replace the force acting at A by a force and couple

(i) at B

(ii) at C



A

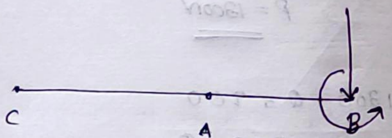
$\Sigma F_V = -10 kN$

$\Sigma F_H = 0$

$\Sigma M_B = -10 \times 3 = -30 \text{ kNm}$

$= 30 \text{ kNm}$

= counter clock wise



B

$\Sigma F_V = -10 kN$

$\Sigma F_H = 0$

$\Sigma M_B = -10 \times 0 = 0$

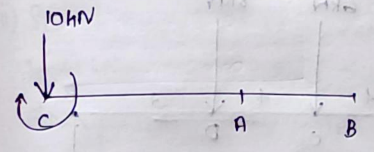
∴ a counter clock wise moment of magnitude 30 kNm should be applied at B.

A

$\Sigma F_V = -10 kN$

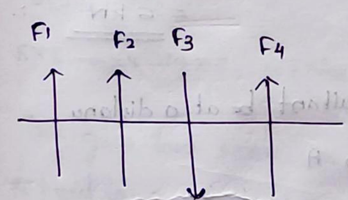
$\Sigma F_H = 0$

$\Sigma M_C = 10 \times 4 = 40 \text{ kNm}$ (c.w)



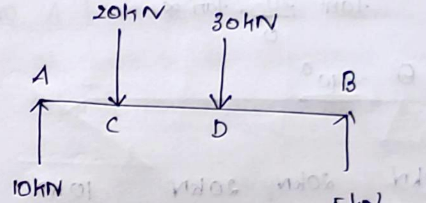
Resultant of Parallel forces.

Since all forces are parallel, resultant will be the algebraic sum of the forces.



$R = F_1 + F_2 - F_3 + F_4$

Q. Calculate the resultant of the force system of parallel forces as shown in figure.

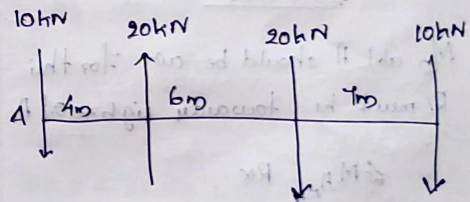


$R = 10 - 20 - 30 + 5$

$= -35 kN$

$= 35 kN \downarrow$

Q. Determine the resultant of the system of forces shown in fig



$\Sigma F_H = 0$

$\Sigma F_V = -10 + 20 - 20 - 10 = -20 kN$

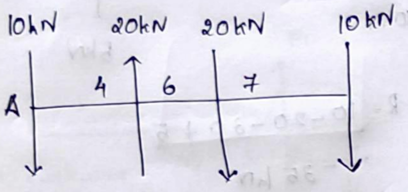
$R = \sqrt{(F_H)^2 + (F_V)^2} = \sqrt{0 + (20)^2}$

$= 20 kN$

$$\theta = \tan^{-1}\frac{\Sigma F_V}{\Sigma F_H}$$

$$= \tan^{-1}\frac{20}{0} = \tan^{-1}\infty$$

$$\theta = 90^\circ$$



$$\Sigma M_A = 10\times0 - 20\times4 + 20\times10 + 10\times17$$

$$= -80 + 200 + 170$$

$$= 290 \text{ kNm} \quad (\text{cw})$$

M_R abt A should be cw ∴ for this R must be towards right of A

$$\Sigma M_A = Rx$$

$$x = \frac{\Sigma M_A}{R} = \frac{29\cancel{0}}{2\cancel{0}} = 14.5$$

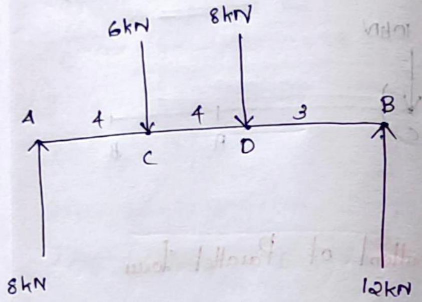
$$x = 14.5$$

Q. A rigid bar AB is acted upon by forces as shown in fig. Reduce the force system to

(i) single force

(ii) force moment system at A &

(iii) force moment system at D.



$$\Sigma F_y = 8 - 6 - 8 + 12 = \cancel{8}\ 6 \text{ kN}$$

$$R = \sqrt{(\Sigma F_h)^2 + (\Sigma F_v)^2} = \sqrt{0+36}$$

$$= 6 \text{ kN}$$

Let resultant be at a distance x from A

$$\Sigma M_B = 6\times4 + 8\times8 + -12\times11$$

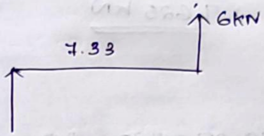
$$= -44 \text{ kNm}$$

$$= 44 \text{ kNm ccw}$$

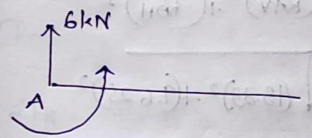
M of resultant abt A = $6 \times x$

$$6 \times x = 44$$

$$x = \frac{44}{6} = 7.33$$



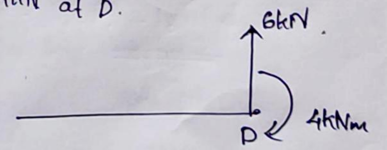
(ii) Sum of moments of forces abt A is 44 kNm. The resultant force is 6kN upwards ∴ the system can be reduced to a force moment system at A as



(iii) The sum of moments of forces about D is

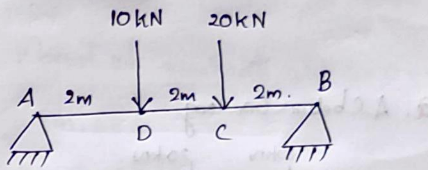
$$8\times8 - 6\times4 - 12\times3 = 4 \text{ kNm cw}$$

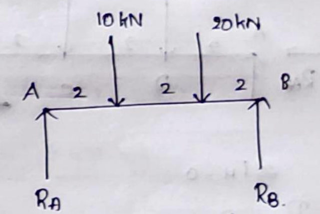
The given force system can be reduced to a single force of 6kN along with a cw moment of 4kN at D.



Simple beams subjected to concentrated vertical loads.

Q. A beam 6m long is loaded as shown in the figure. Calculate the reactions at A & B.





$$\Sigma F_V = 0$$

$$R_A - 10 - 20 + R_B = 0$$

$$R_A + R_B = 30 \longrightarrow ①$$

$$\Sigma M_A = 0$$

$$10\times2 + 20\times4 + -R_B\times6 = 0$$

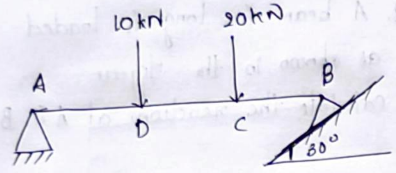
$$R_B = \frac{100}{6} = 16.67 \text{ kN}$$

$$R_A + R_B = 30$$

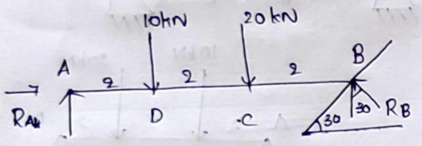
$$R_A = 30 - R_B = 30 - 16.67$$

$$= 13.33 \text{ kN}$$

Q. A beam 6m long is located as shown in fig. calculate the reactions at A & B.



Q. A beam 6m long



$\Sigma F_H = 0$

$R_{AH} - R_B \sin 30 = 0.$

$R_{AH} = R_B \sin\theta \rightarrow ①$

$\Sigma F_V = 0$

$R_{AV} - 10 - 20 + R_B \cos 30 = 0$

$R_{AV} + R_B \cos\theta = 30 \rightarrow ②$

$\Sigma M_A = 0$

$10\times2 + 20\times4 - (R_B \cos 30)\times6 = 0$

$20 + 80 = 6 R_B \cos 30$

$R_B = \dfrac{100}{6\cos 30}$

$= \underline{19.25 \text{ kN}}$

From ①

$R_{AH} = R_B \sin 30$

$= 19.25 \times 0.5$

$= \underline{\underline{9.625 \text{ kN}}}$

From ②

$R_{AV} + 19.25 \cos 30 = 30$

$R_{AV} = 80 - 19.25 \cos 30$

$= \underline{\underline{13.33 \text{ kN}}}$

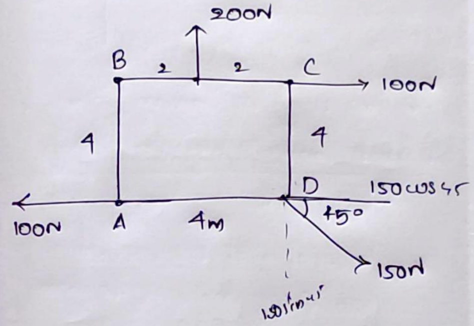
$R_A = \sqrt{(R_{AV})^2 + (R_{AH})^2}$

$= \sqrt{(13.33)^2 + (9.625)^2}$

$= \underline{\underline{16.44 \text{ kN}}}$

Miscellaneous Questions

Q. For the system of forces shown in figure, determine the magnitude, direction and position of the resultant force w.r.t A



Resolving force horizontally

$\Sigma F_H = 100 - 100 + 150 \cos 45$

$= \underline{\underline{106.07 \text{N}}}$

$\Sigma F_V = 200 - 150 \sin 45$

$= \underline{\underline{93.93 \text{N}}}$

$R = \sqrt{(106.07)^2 + (93.93)^2}$

$= \underline{\underline{141.68 \text{N}}}$

$\theta = \tan^{-1} \dfrac{93.93}{106.07}$

$= \underline{\underline{41.53°}}$

Sum of moments of all forces abt A

$\Sigma M_A = 150 \sin 45 \times 4 + 100\times4 - 200\times2$

$= \underline{\underline{424.26 \text{ Nm}}}$ c.w.

Moment of resultant abt $A = R\times x$

$= 141.68x$

Equating ΣM_A & Rx

$424.26 = 141.68x$

$x = \dfrac{424.26}{141.68}$

$= \underline{\underline{3\text{m}}}$