MODULE - 1

Introduction To Mechanics

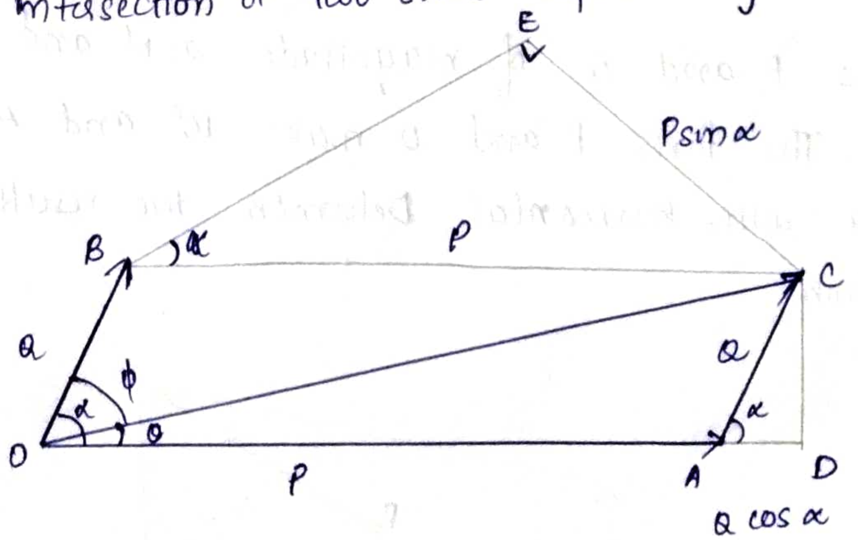
Mechanics

- Statics
  - study of particle and rigid bodies at rest
- Dynamics
  - Kinematics
  - Kinetics

STATICS

PARALLELOGRAM LAW

If two forces acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of llgm which passes through the point of intersection of two sides representing the forces.



$$OC^2 = OD^2 + DC^2$$

$$= (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

$$= P^2 + 2PQ\cos\alpha + Q^2\cos^2\alpha + Q^2\sin^2\alpha$$

$$= P^2 + Q^2 + 2PQ\cos\alpha$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha \Rightarrow \boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}}$$

$$\boxed{\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P + Q\cos\alpha} = \frac{\sin\alpha}{\frac{P}{Q} + \cos\alpha}}$$

case (i) $\alpha = 0°$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{P^2 + Q^2 + 2PQ}$$

$$R = (P + Q)$$

case (ii) $\alpha = 90°$

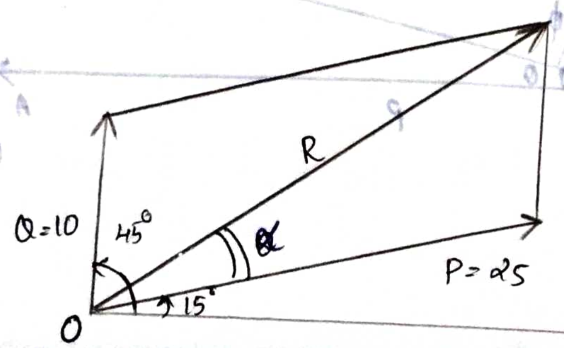
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90°} = \sqrt{P^2 + Q^2}$$

case (iii) $\alpha = 180°$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180°} = \sqrt{P^2 + Q^2 - 2PQ}$$

$$R = P - Q$$

(1) Two forces P and Q of magnitude 25N and 10N are acting at a point. The forces P and Q make 15° and 45°, measured anticlockwise with horizontal. Determine the resultant in magnitude and direction.



$P = 25N$ $\quad Q = 10N$ $\quad \alpha = 45 - 15 = 30^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{25^2 + 10^2 + 2 \times 25 \times 10 \times \cos 30^\circ}$$

$$= \sqrt{725 + 500\cos 30^\circ} = \underline{34.03N}$$

The inclination of resultant force with direction of force P

$$\theta = \tan^{-1} \frac{\sin\alpha}{\cos\alpha + \frac{P}{Q}} = \tan^{-1} \frac{\sin 30^\circ}{\cos 30^\circ + 2.5}$$

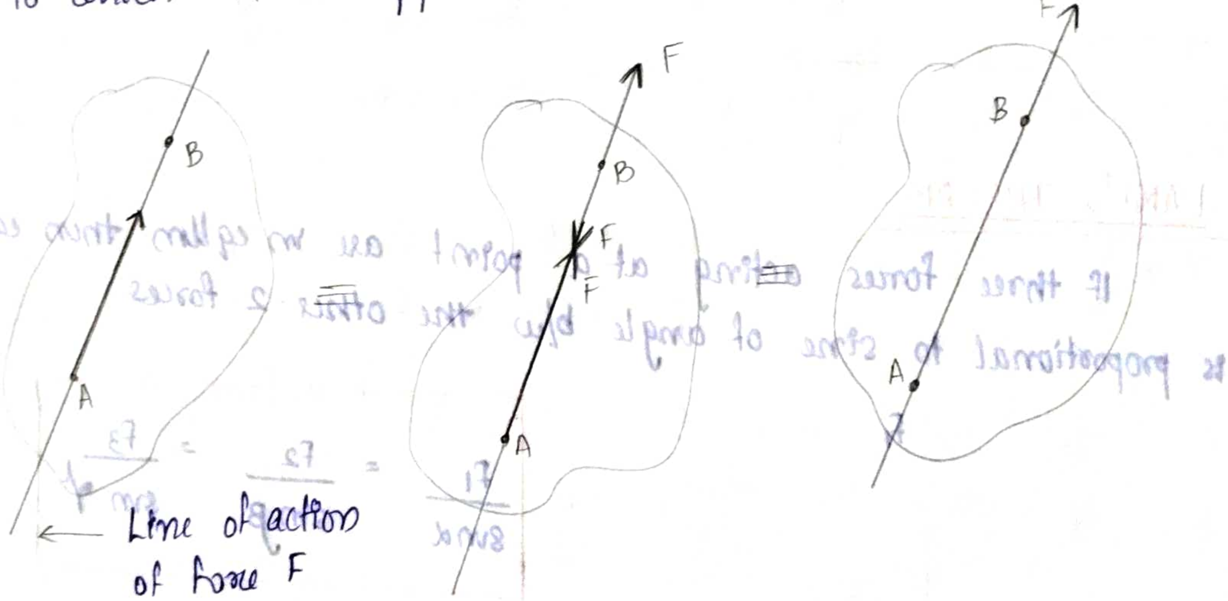
$$\theta = \underline{8.45^\circ}$$

$\therefore$ Inclination with horizontal is $15^\circ + \theta$

$$= 15 + 8.45 = \underline{23.45^\circ}$$

PRINCIPLE OF TRANSMISSIBILTY

The point of application of force can be transmitted along its line of action without changing the effect of the force on any rigid body to which its is applied.



EQUILIBRIUM LAWS

Equilibrium :- Condition in which the resultant of all forces & moments acting on the body is zero. $F = 0$, $M = 0$

For a two body force system

Forces must be

→ Equal in magnitude

→ opp. in direction

→ collinear

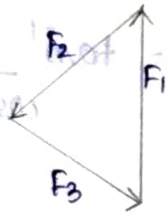
• For a 3 force body system

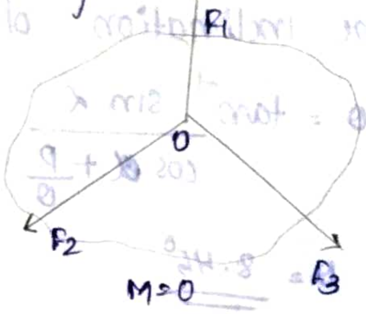
Forces must be

→ concurrent

→ line of action should meet at a point

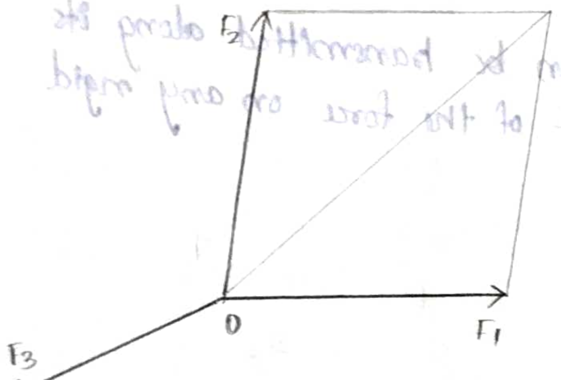
→ Sum of any 2 force = 3rd force

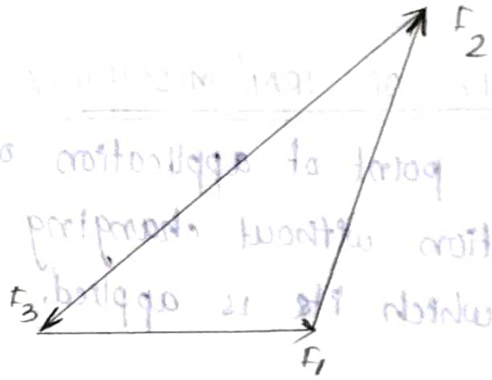




LAW OF TRIANGLE OF FORCES

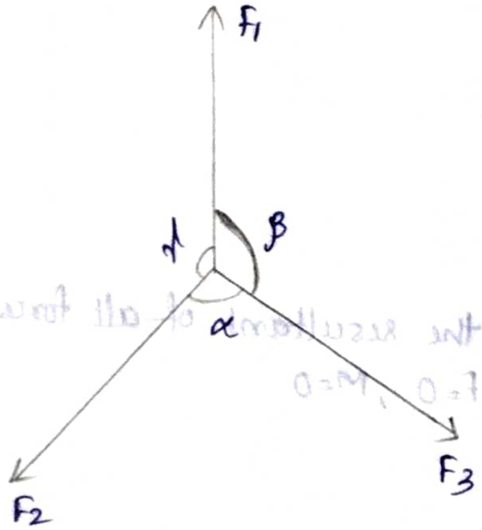
If three coplanar forces acting at a point are in eqllm, then they can be represented in magnitude and direction by the sides of a Δ taken in same order.





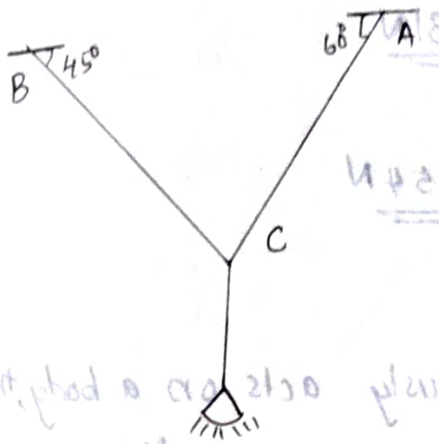
LAMI'S THEORM

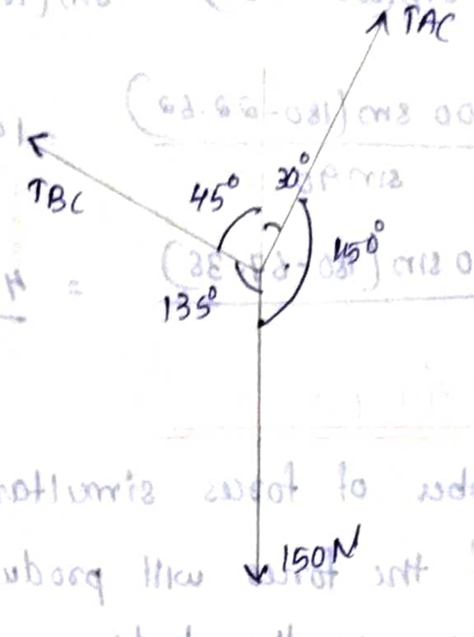
If three forces acting at a point are in eqllm then each force is proportional to sine of angle b/w the other 2 forces



$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

(2) An electric light fixture weighing 150N hangs from a point C by two stay wires AC and BC as shown. Determine tensions in AC and BC using Lami's



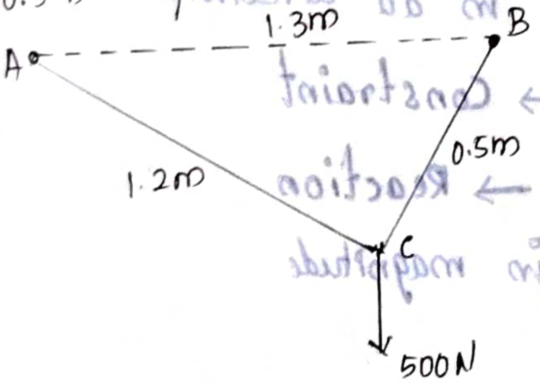


Using Lami's theorm,

$$\frac{150}{\sin 75} = \frac{T_{BC}}{\sin 150^\circ} = \frac{T_{AC}}{\sin 135^\circ}$$

$$T_{BC} = \frac{150 \times \sin 150}{\sin 75} = 77.65N \qquad T_{AC} = \frac{150 \times \sin 135}{\sin 75} = 109.81N$$

(2) Two cables AC and BC are tied together at the point C to support a load of 500N at C. A and B are at a distance of 1.3m and are on the same horizontal plane. AC and BC are 1.2m and 0.5m respectively. Find the tensions in cable AC and BC.



$$AC^2 + BC^2 = AB^2$$
$$= 1.2^2 + 0.5^2 = 1.3^2$$

$\because AC^2 + BC^2 = AB^2$, angle $ACB = 90^\circ$ $[\theta + \phi = 90^\circ]$

$$\theta = \cos^{-1}\frac{1.2}{1.3} = 22.62^\circ \qquad \phi = 90 - \theta = 90 - 22.62 = 67.38^\circ$$

Applying Lami's theorem

$$\frac{500}{\sin(\phi + \theta)} = \frac{T_{AC}}{\sin(180 - \theta)} = \frac{T_{BC}}{\sin(180 - \phi)}$$

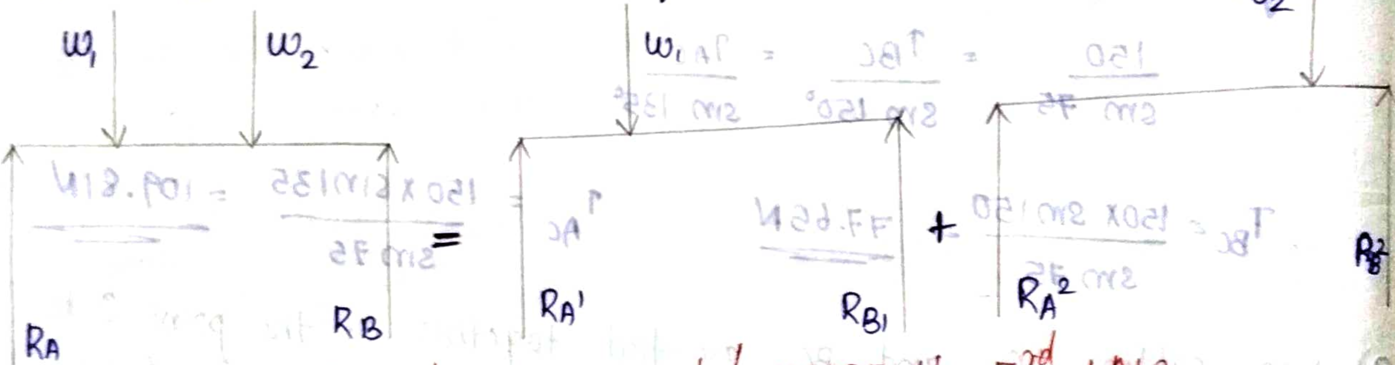
$$\frac{500}{\sin 90} = \frac{T_{AC}}{\sin(180-22.62)} = \frac{T_{BC}}{\sin(180-67.38)}$$

$$T_{AC} = \frac{500 \sin(180-22.62)}{\sin 90} = \underline{192.31N}$$

$$T_{BC} = \frac{500 \sin(180-67.38)}{} = \underline{461.54N}$$

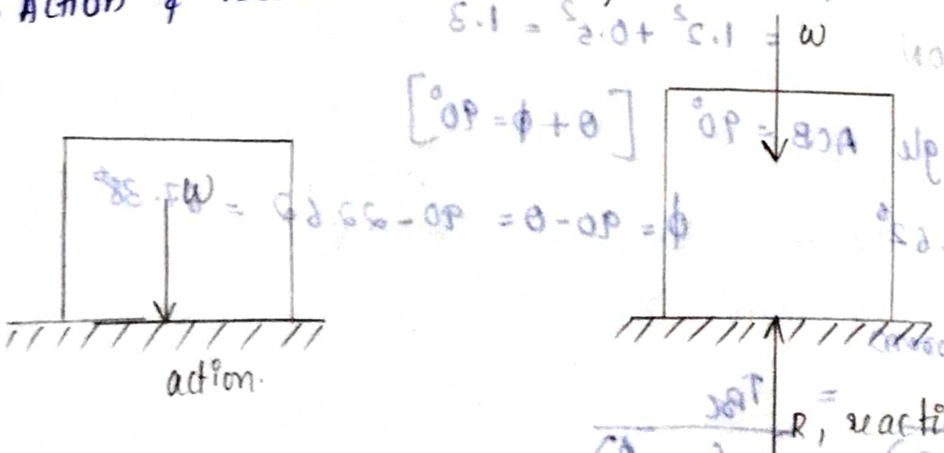
PRINCIPLE OF SUPERPOSITION

If a number of forces simultaneously acts on a body, then each ~~other~~ one of the forces will produce the same effect, when this force acts alone in the body.

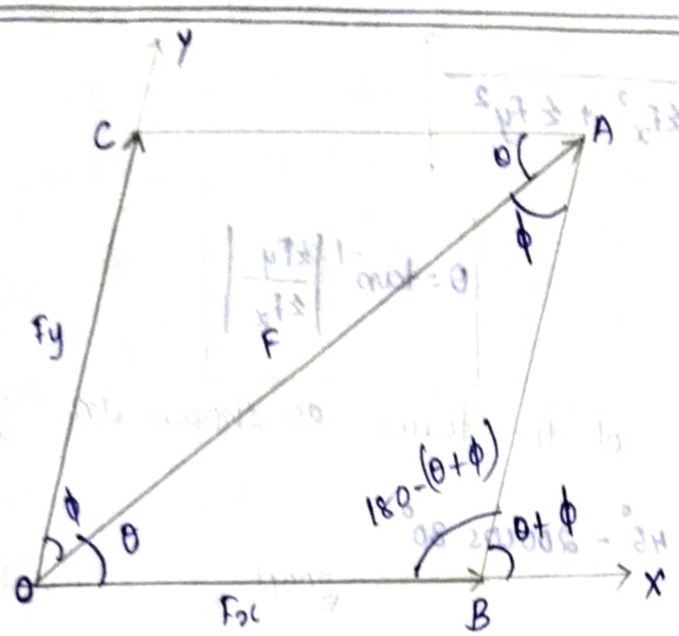


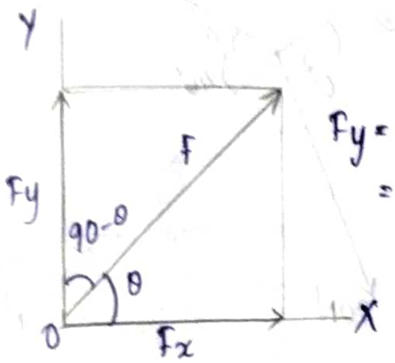
LAWS OF ACTION AND REACTION / NEWTON'S 3rd LAW

- Newton's third law :- To every action, there is an equal & opp reaction.
- A body is not always free to move in all directions
- Restriction to free motion of a body → **Constraint**
- Force applied by a surface on the body → **Reaction**
- Action & reaction are always equal in magnitude



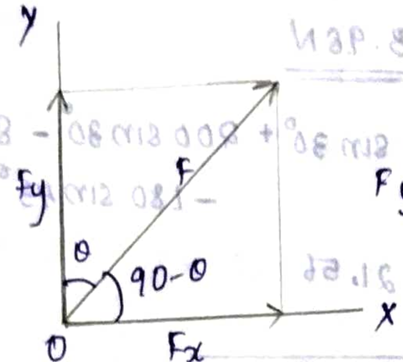
COMPOSITION & RESOLUTION OF FORCES





$F_y = F\cos(90-\theta)$

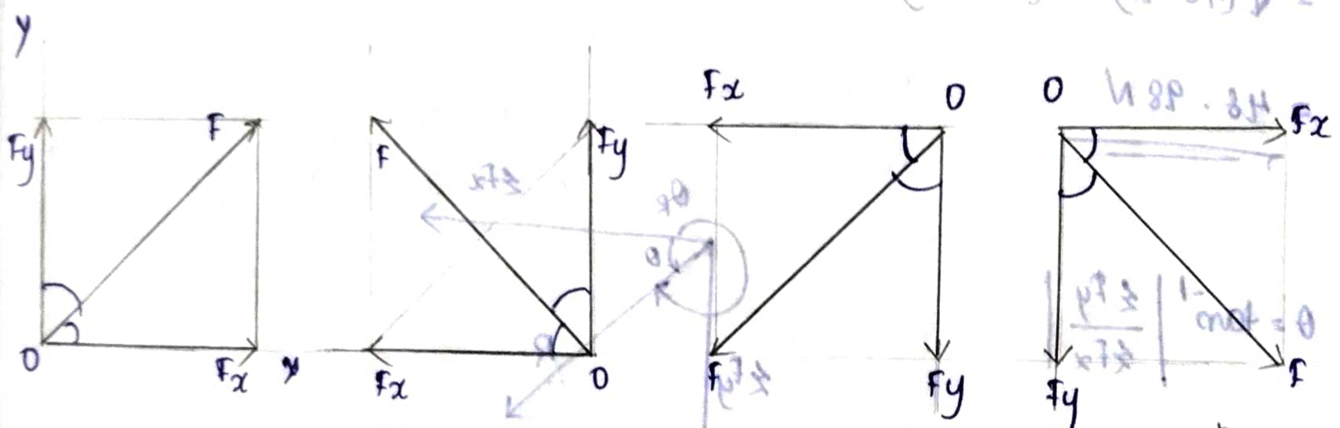
$= F\sin\theta$



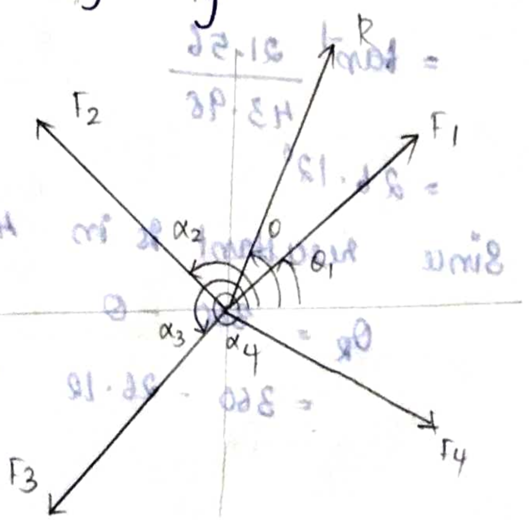
$F_y = F\cos\theta$

$F_x = F\,\cancel{\sin}\cos(90-\theta)$

$= F\sin\theta$



RESULTANT OF CO-PLANAR CONCURRENT FORCES



$$F_1\cos\alpha_1 + F_2\cos\alpha_2 + F_3\cos\alpha_3 + F_4\cos\alpha_4 = R\cos\theta$$

$$\Sigma F_x = R\cos\theta$$

$$F_1\sin\alpha_1 + F_2\sin\alpha_2 + F_3\sin\alpha_3 + F_4\sin\alpha_4 = R\sin\theta$$

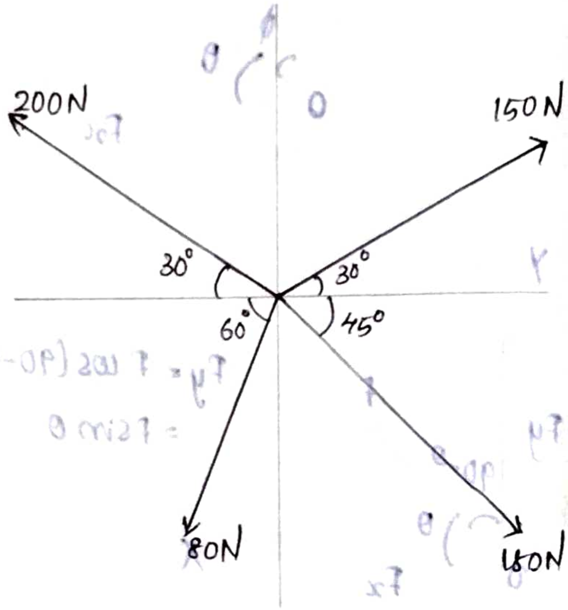
$$\Sigma F_y = R\sin\theta$$

$$\Sigma F_x^2 + \Sigma F_y^2 = R^2\cos^2\theta + R^2\sin^2\theta = R^2(\sin^2\theta + \cos^2\theta)$$

$$\boxed{R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}}$$

$$\tan\theta = \frac{|\Sigma F_y|}{|\Sigma F_x|} \qquad \boxed{\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|}$$

(3) Find the resultant of the forces as shown in fig.



$$\Sigma F_x = 150\cos 30^\circ + 180\cos 45^\circ - 200\cos 30^\circ - 80\cos 60^\circ$$

$$= \underline{43.98\,N}$$

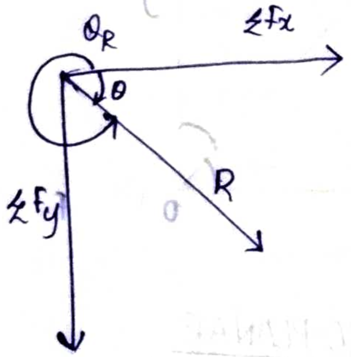
$$\Sigma F_y = 150\sin 30^\circ + 200\sin 30^\circ - 80\sin 60^\circ - 180\sin 45^\circ$$

$$= -21.56$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$= \sqrt{(43.98)^2 + (-21.56)^2}$$

$$= \underline{48.98\,N}$$



$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\frac{21.56}{43.98}$$

$$= 26.12^\circ$$

Since resultant is in 4th quadrant, the inclination of resultant

$$\theta_R = 360 - \theta$$

$$= 360 - 26.12 = \underline{333.88^\circ}$$

06/05/21

EQUILIBRIUM EQUATIONS

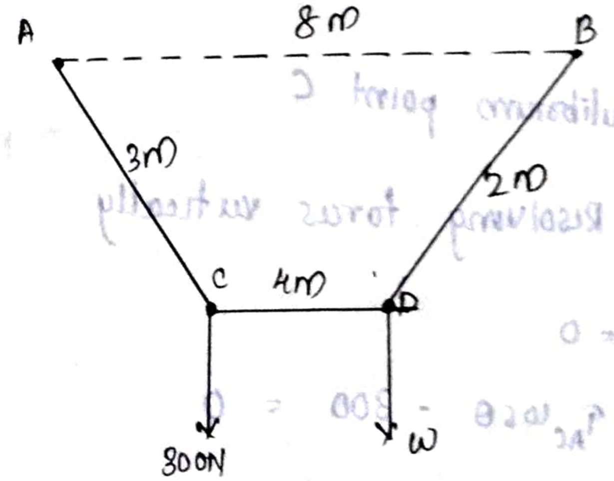
- A particle / body is in equilibrium if the resultant of no of forces acting on it is zero.
- If $R \neq 0$, Force required to bring the body to rest :- EQUILIBRIUM
- Resultant & Equilibrium are in equal in magnitude and opposite in direction

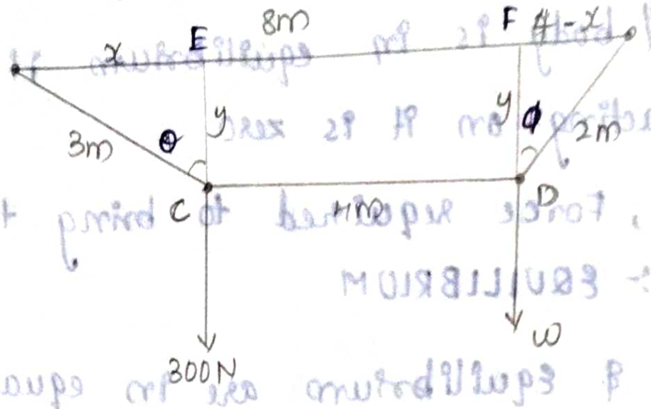
$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$, where ΣF_x and ΣF_y are the sum of components of all the forces along two mutually $\perp^r$ x and y directions.

For R to be zero, both ΣF_x and ΣF_y must be zero. therefore the equation of equilibrium are,

1) $\Sigma F_x = 0$ 2) $\Sigma F_y = 0$

(1) A rope 9m long is connected at A and B, two points on same level 8m apart. A load of 300N is suspended from a point fra C on rope 3m from A. What load connected to a point D, on the rope, 2m from B is necessary to keep CD parallel to AB.





From ΔAEC,

$$y^2 = 3^2 - x^2$$

From ΔBDF

$$y^2 = 2^2 - (4-x)^2$$

$$3^2 - x^2 = 2^2 - (16 + x^2 - 8x)$$

$$9 - x^2 = 4 - 16 - x^2 + 8x \Rightarrow 8x = 21 \Rightarrow x = 2.625m$$

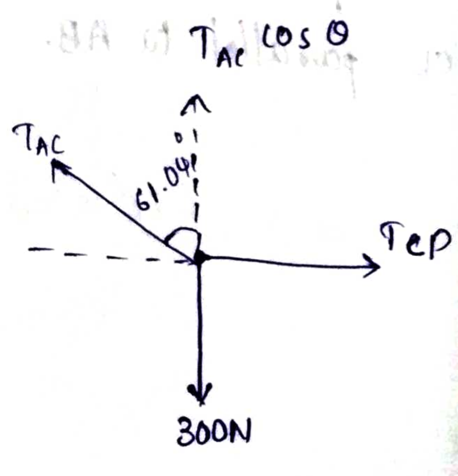
$$\sin\theta = \frac{x}{3} = \frac{2.625}{3} = 0.875$$

$$\theta = \sin^{-1} 0.875 = 61.04^\circ$$

$$\sin\phi = \frac{4-x}{2} = \frac{4-2.625}{2} = \frac{1.375}{2} = 0.6875$$

$$\phi = \sin^{-1} 0.6875 = 43.43^\circ$$

At equilibrium point C



Resolving forces vertically

$$\Sigma F_v = 0$$

$$T_{AC}\cos\theta - 300 = 0$$

$$T_{AC} = \frac{300}{\cos\theta} = \frac{300}{\cos 61.04} = \frac{300}{0.484} = \underline{619.83\,N}$$

Resolving forces horizontally,

$$\Sigma F_H = 0$$

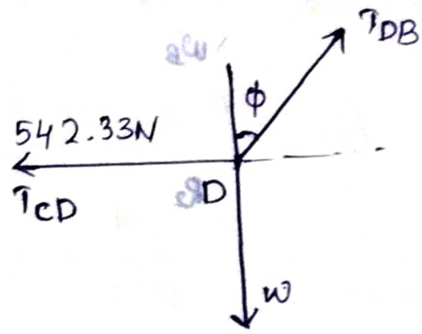
$$T_{CD} - T_{AC}\sin\theta = 0$$

$$T_{CD} = T_{AC}\sin\theta = 619.83 \sin 61.04$$

$$= 619.88 \times 0.875$$

$$= \underline{542.33\,N}$$

Consider equilibrium point at D



$$\Sigma F_H = 0$$

$$T_{DB}\sin\phi - T_{CD} = 0$$

$$T_{DB} = \frac{T_{CD}}{\sin\phi} = \frac{542.33}{\sin 43.43}$$

$$= \frac{542.33}{0.687} = \underline{788.56\,N}$$

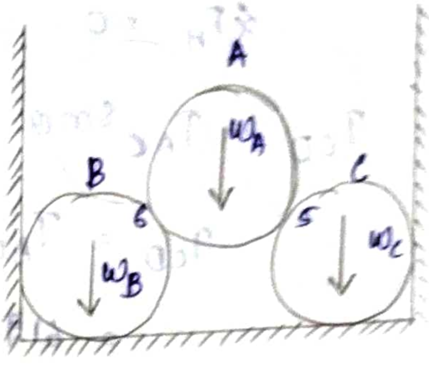
$$\Sigma F_V = 0$$

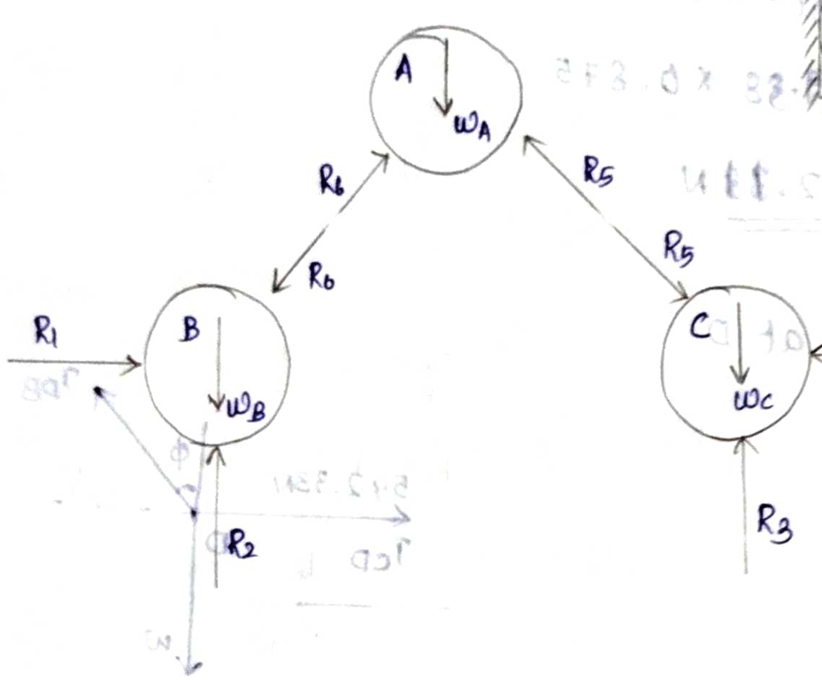
$$T_{DB}\cos\phi - W = 0$$

$$W = T_{DB}\cos\phi = 788.56 \cos 43.43$$

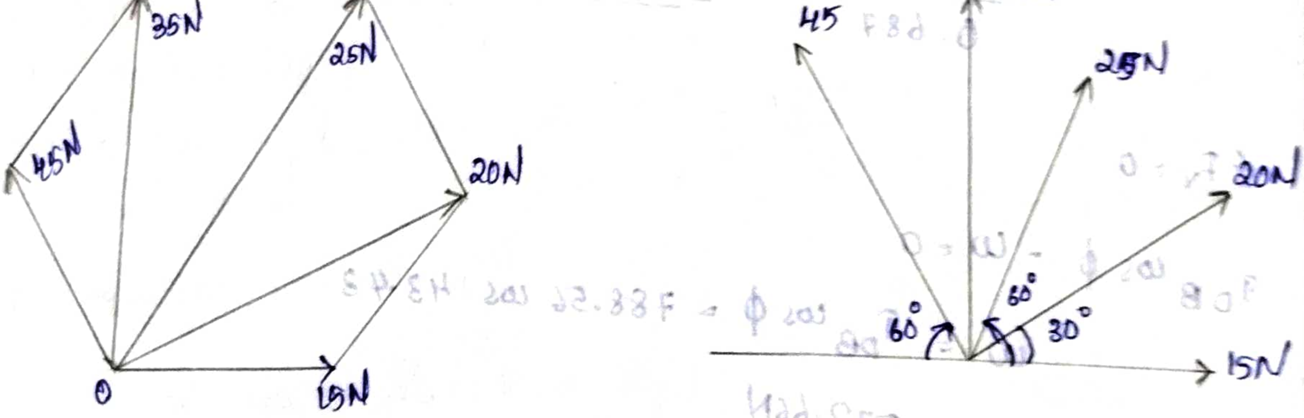
$$= \underline{572.66N}$$

(5) Three smooth identical spheres A, B, C are placed in a rectangular channel as shown in fig. Draw the free body diagram of each sphere. (KTU June 2016)





(6) Forces of 15N, 20N, 25N, 35N and 45N act at an angular point of regular hexagon towards the other angular points as shown in Fig. Calculate the magnitude and direction of the resultant force. (KTU Aug 2016)



Resolving forces along x-axis

$$\Sigma F_x = 15 + 20\cos 30^\circ + 25\cos 60^\circ + 0 - 45\cos 60^\circ$$

$$= \underline{22.32\,N}$$

Resolving the forces along y-axis

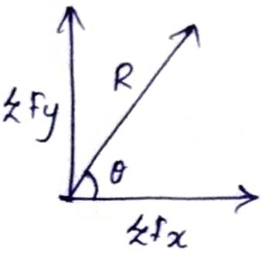
$\Sigma F_y = 0 + 20\sin 30 + 25\sin 60 + 35 + 45\sin 60 = \underline{105.62N}$

Resultant $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$= \sqrt{22.32^2 + 105.62^2} = \underline{107.95\ N}$

Inclination of resultant with horizontal

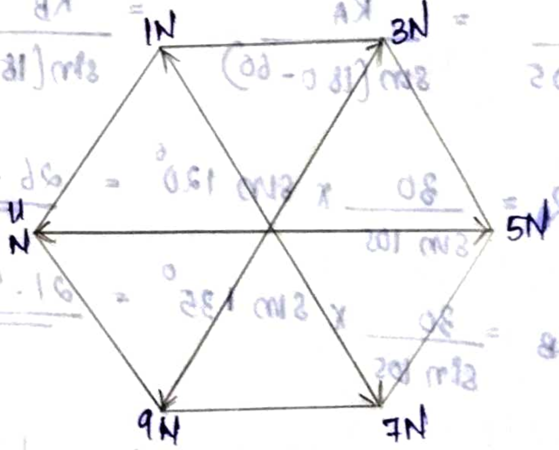
$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left|\frac{105.62}{22.32}\right| = \underline{78.07^\circ}$



Inclination of resultant $\theta_R = \theta = \underline{78.07^\circ}$

(7) Concurrent forces 1, 3, 5, 7, 9, 11 are applied to the center of a regular hexagon acting towards its vertices as shown in Fig. Determine the magnitude and direction of resultant (KTU, July 2016)

In a regular hexagon, the angle substended by each side at the center is 60°



Resolving the forces horizontally,

$\Sigma F_H = 5 + 3\cos 60^\circ - 1\cos 60^\circ - 11 - 9\cos 60^\circ + 7\cos 60^\circ$

$= -6N$

$= 6N$ towards left

ΣF_V [Resolving the forces vertically]

$\Sigma F_V = 0 + 3\sin 60^\circ + 1\sin 60^\circ + 0 - 9\sin 60^\circ - 7\sin 60^\circ$

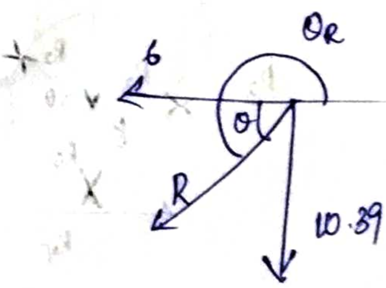
$= -10.39N$

Resultant $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{6^2 + (10.39)^2} = \underline{12N}$

Inclination of resultant with horizontal

$\tan\theta = \left|\frac{\Sigma F_V}{\Sigma F_H}\right| = \frac{10.39}{6} = 1.732$

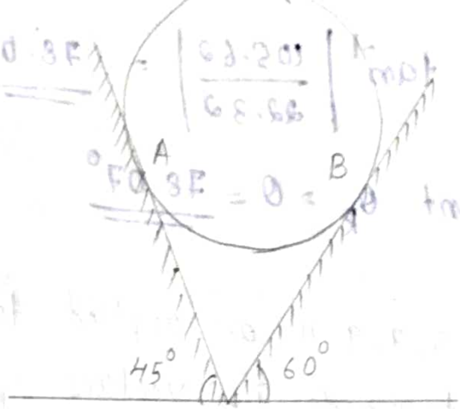
$\theta = \tan^{-1} 1.732 = \underline{60^\circ}$



Since the resultant is in third quadrant,

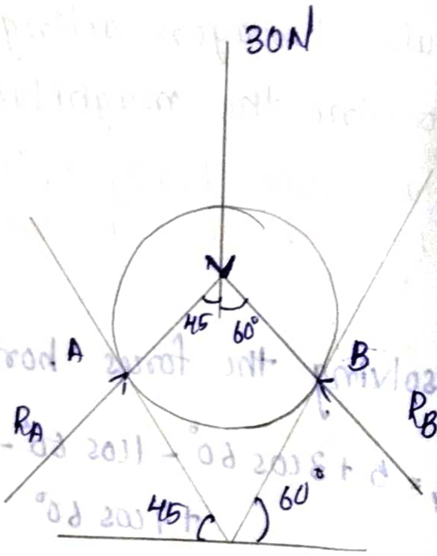
$$\theta_R = 180^\circ + \theta = 180 + 60 = 240^\circ$$

(8) A solid cylinder 30mm diameter and weighing 300N is placed in a triangular channel as shown. Neglecting the friction b/w the contact surfaces, calculate the normal reaction on the sides of B channel. (KTU May 2019)



Applying Lami's theorem,

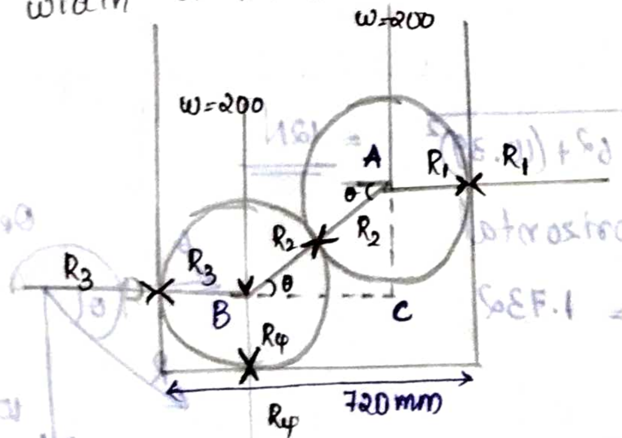
$$\frac{30}{\sin 105} = \frac{R_A}{\sin(180-60)} = \frac{R_B}{\sin(180-45)}$$



$$R_A = \frac{30}{\sin 105} \times \sin 120^\circ = \underline{26.89 N}$$

$$R_B = \frac{30}{\sin 105} \times \sin 135^\circ = \underline{21.96 N}$$

(9) Two smooth cylinders A and B each of diameter 400mm & weight 200N rest in a horizontal channel having vertical walls and base width of 720mm as shown in fig. Find reaction at P, Q, R.



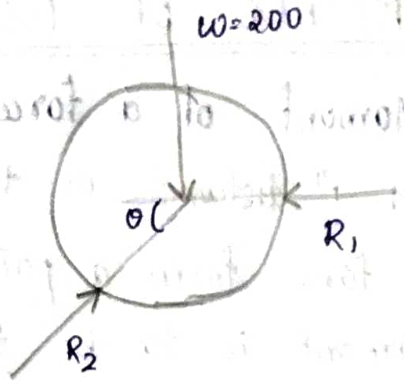
AB - 400mm

BC = 720 - 400 = 320mm

$$\cos\theta = \frac{BC}{AB} = \frac{320}{400}$$

$$\underline{\theta = 36.87^\circ}$$

Consider equilibrium of upper cylinder



Resolving the force vertically

For $\Sigma F_v = 0$

$R_2 \sin\theta - 200 = 0$

$$R_2 = \frac{200}{\sin 36.87} = \underline{333.33\,N}$$

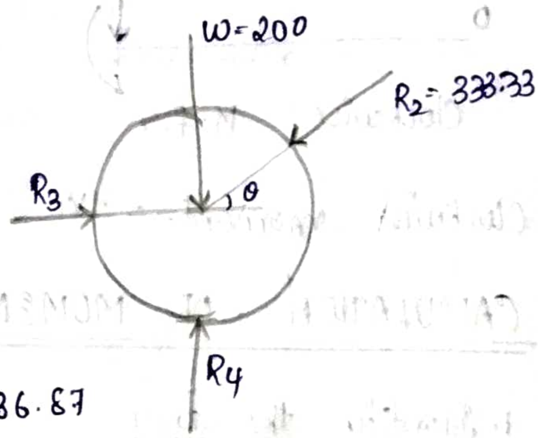
Resolving forces horizontally

$R_2 \cos\theta - R_1 = 0$

$R_1 = R_2 \cos\theta$

$= 333.33 \times \cos 36.87 = \underline{266.67\,N}$

Consider equilibrium of lower cylinder



Resolving the force vertically,

$\Sigma F_V = 0$

$R_4 - 200 - 333.33 \sin\theta = 0$

$R_4 = 200 + 333.33 \sin 36.87$

$= \underline{400\,N}$

Resolving the force horizontally,

$\Sigma F_H = 0$

$R_3 - 333.33 \cos\theta = 0$

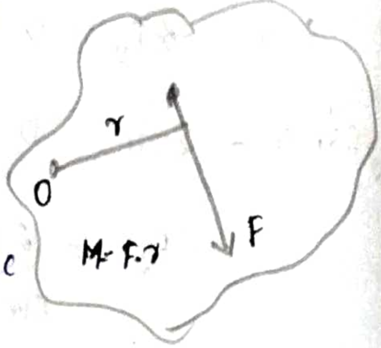
$R_3 = 333.33 \cos 36.87$

$= \underline{266.67\,N}$

Reaction at P = 266.67 N
Reaction at Q = 400 N
Reaction of R = 266.67 N

METHOD OF MOMENTS

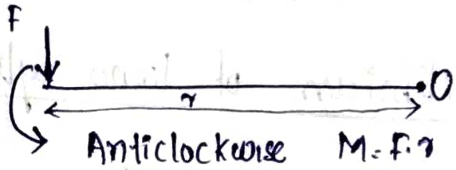
- **Moment of a force about a point** : Product of a force and ⊥ distance of the line of action of the force from a point about which moment is to be taken.



- Moment :- Rotating effect produced by the force on the body about that point.
- Perpendicular distance = Arm of the force / Moment arm
- Point about which moment is taken = Moment centre

CLOCKWISE & ANTICLOCKWISE MOMENT

Clockwise $M = F.r$



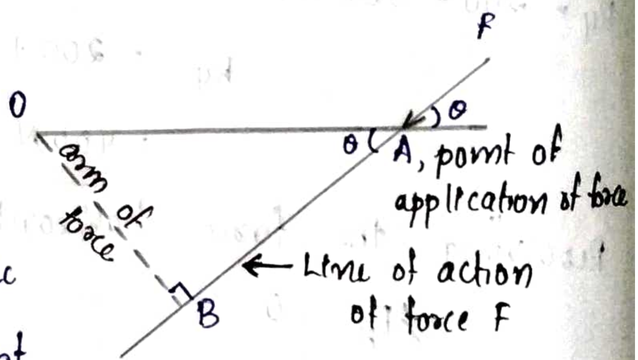
Anticlockwise $M = F.r$

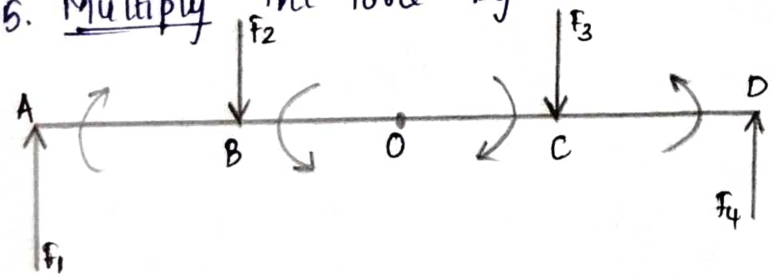
Clockwise moment = +ve

Anticlockwise moment = −ve

CALCULATION OF MOMENTS

1. Identify the force
2. Identify the moment centre
3. Identify the line of action of force
4. Calculate the ⊥ distance of the line of action of the force from moment centre.
5. Multiply the force by the calculated ⊥ distance





- Moment of force F_1 about O $= F_1 \times OA$; C.W
- Moment of force F_2 about O $= F_2 \times OB$, C.C.W
- Moment of force F_3 about O $= F_3 \times OC$, C.W
- Moment of force F_4 about O $= F_4 \times OD$, C.C.W

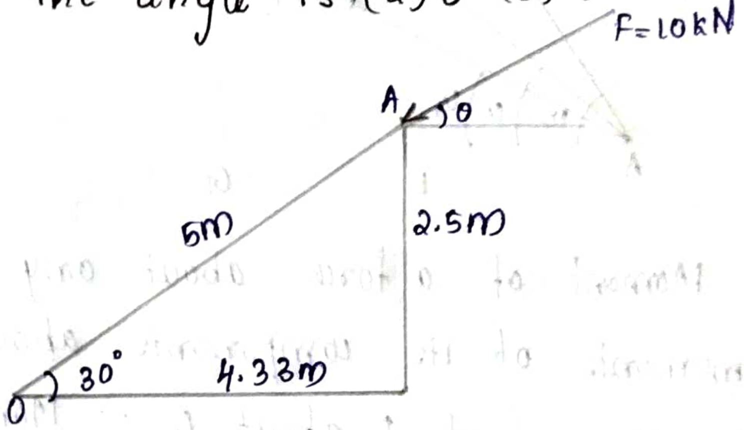
MAX & MIN MOMENTS

- Maximum moment :- Line of action of force is ⊥ to the line joining the moment centre & point of application of force
- Minimum moment :- (M=0) when,

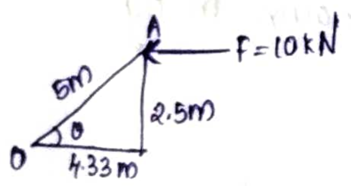
  (i) The force acts at the moment centre itself

  (ii) When the line of action of the force passes through the moment centre.

(10) Calculate the moment of force F = 10 kN acting at point A as shown in the fig when the angle is (a) 0° (b) 30° (c) 90°

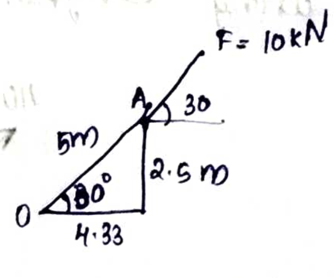


(a) 0°



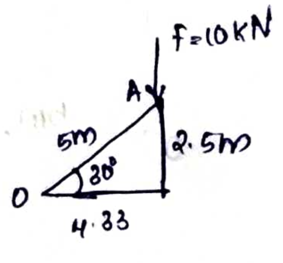
Moment M = 10 × 2.5 = 25 kN-m (anticlockwise)

(b) 30°



Moment M = 10 × 0 = 0 [same line of action]

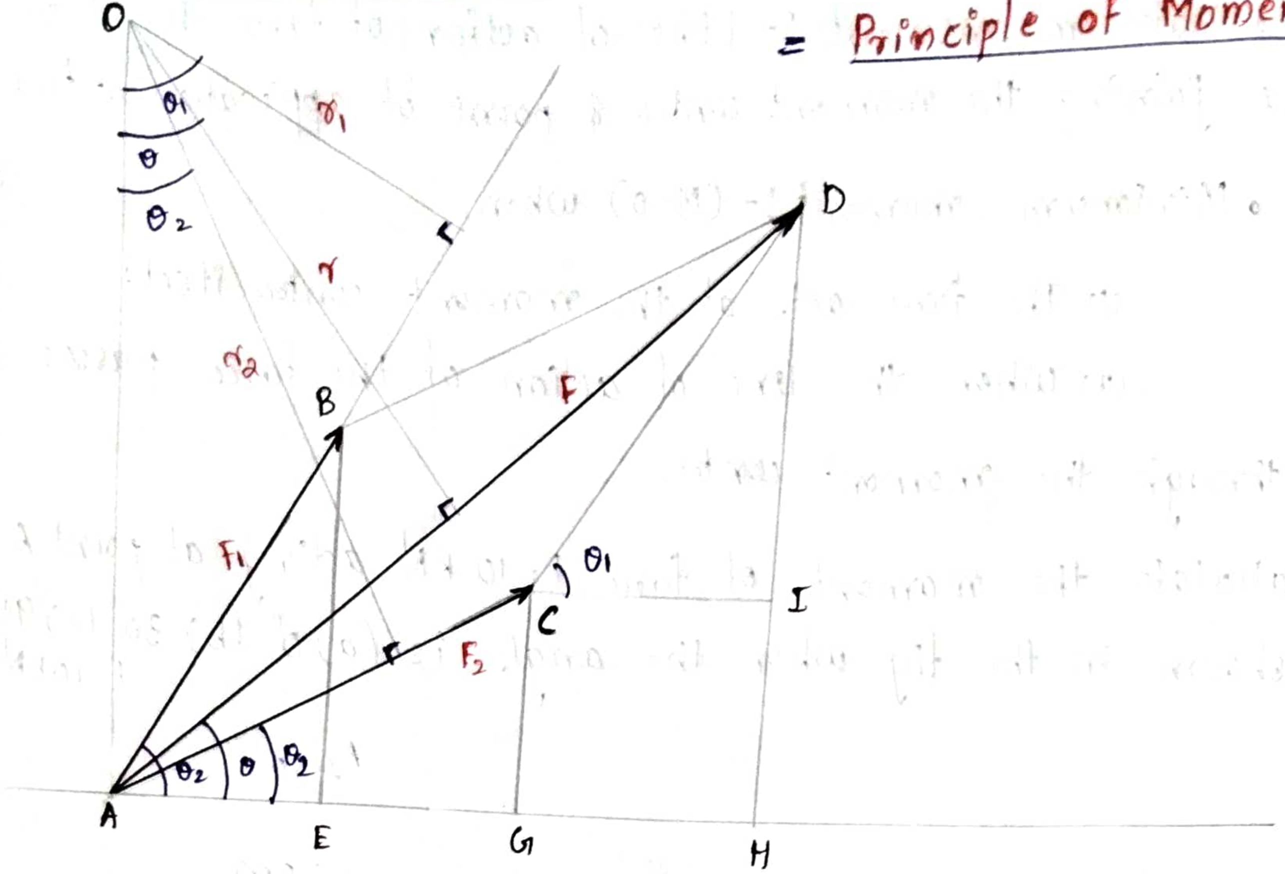
(c) 90°



Moment M = 10 × 4.33 = 43.3 kN-m (clockwise)

VARIGNON'S THEOREM OF MOMENTS

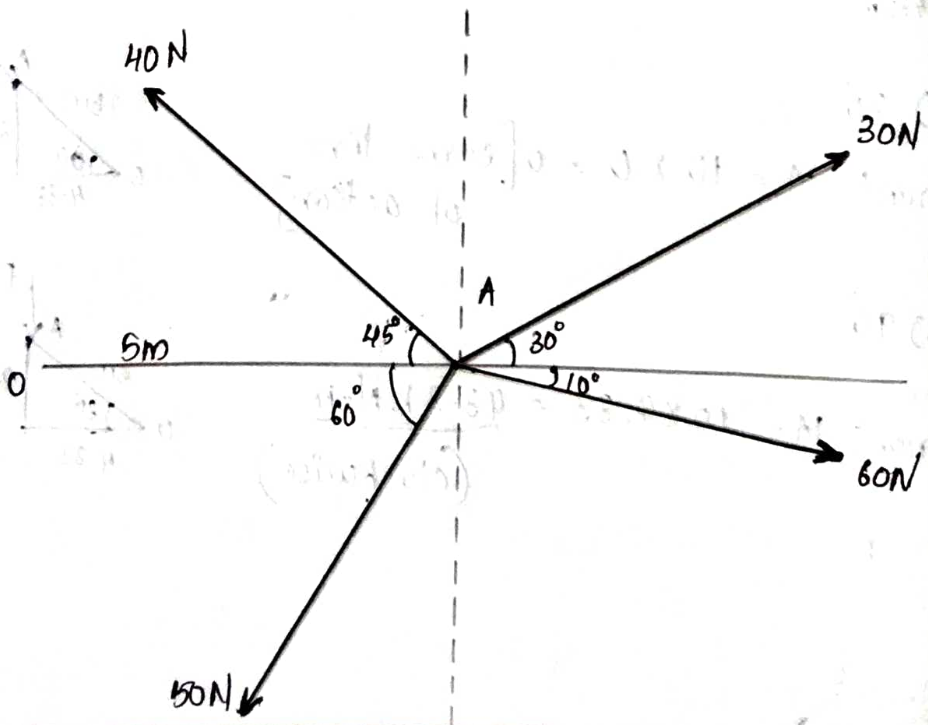
= Principle of Moments



Moment of a force about any axis is equal to the sum of moments of its components about that axis

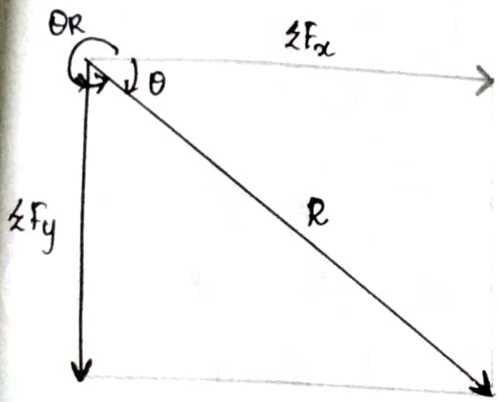
Moment of F about O = Moment of F_1 about O + Moment of F_2 about O

(11) Calculate the moment of the force system shown in fig about O, using varignon's principle

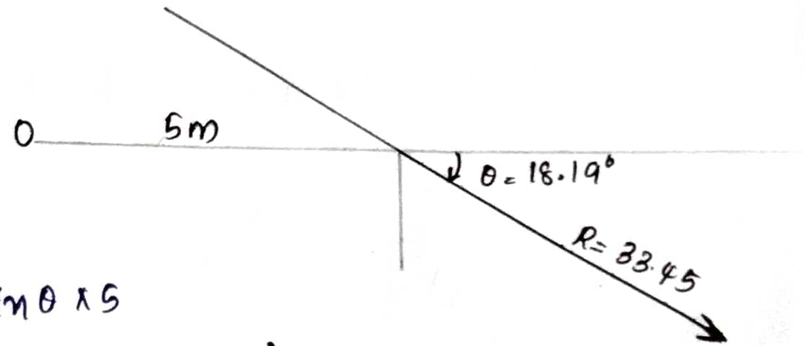


$$\Sigma F_x = 30\cos 30^\circ + 60\cos 10^\circ - 40\cos 45^\circ - 50\cos 60$$
$$= \underline{31.78\,N}$$

$$\Sigma F_y = 30\sin 30^\circ + 40\sin 45^\circ - 50\sin 60^\circ - 60\sin 10^\circ$$
$$= \underline{-10.44\,N}$$



$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left|\frac{10.44}{31.78}\right| = \underline{18.19^\circ}$$



$$M_o = R\sin\theta \times 5$$
$$= 33.45 \times \sin(18.19) \times 5$$
$$= 33.45 \times 0.312 \times 5 = \underline{52.21\,Nm}$$