

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester B.Tech Degree Regular and Supplementary Examination December 2020 (2019 Scheme)

Course Code: MAT101**Course Name: LINEAR ALGEBRA AND CALCULUS****(2019 Scheme)**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 3 marks.*

- 1 Determine the rank of the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$ (3)
- 2 Show that the quadratic form $4x^2 + 12xy + 13y^2$ is positive definite. (3)
- 3 If $z = \sin(y^2 - 4x)$ find the rate of change of z with respect to x at the point $(3, 1)$ with y held fixed. (3)
- 4 Find $\frac{dz}{dt}$ by chain rule, where $z = 3x^2 y^2$, $x = t^4$, $y = t^3$ (3)
- 5 Find the mass of the lamina with density function x^2 which is bounded by $y = x$ and $y = x^2$. (3)
- 6 Evaluate $\iint_R y^2 x \, dA$ over the region $R = \{(x, y), -3 \leq x \leq 2, 0 \leq y \leq 1\}$ (3)
- 7 Test the convergence of the series $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$ (3)
- 8 Does the series $\sum_{k=1}^{\infty} \left(\frac{-3}{4}\right)^k$ converge? If so, find the sum. (3)
- 9 Find the binomial series for $f(x) = (1+x)^{1/3}$ up to third degree term. (3)
- 10 Find the Maclaurin's series of $f(x) = \log(1+x)$ up to third degree term. (3)

PART B*Answer one full question from each module, each question carries 14 marks***Module-I**

- 11 a) Solve the following linear system of equations using Gauss elimination method. $x + y + z = 6$, $x + 2y - 3z = -4$, $-x - 4y + 9z = 18$ (7)

- b) Find eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \quad (7)$$

- 12 a) Show that the equations:

$$x + y + z = a, \quad 3x + 4y + 5z = b, \quad 2x + 3y + 4z = c$$

(i) have no solution if $a = b = c = 1$ (7)

(ii) have many solutions if $a = \frac{b}{2} = c = 1$

- b) Find the matrix of transformation that diagonalize the matrix (7)

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}. \text{ Also, find the diagonal matrix.}$$

Module-II

- 13 a) Find the local linear approximation of $\frac{4y}{x+z}$ at $(1,1,1)$ (7)

- b) Find the absolute extrema of the function $f(x, y) = x^2 - 3y^2 - 2x + 6y$ over the square region with vertices $(0,0), (0,2), (2,2)$ and $(2,0)$. (7)

- 14 a) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$. (7)

- b) Locate all relative extrema of $f(x, y) = 2xy - x^3 - y^2$ (7)

Module-III

- 15 a) Use double integrals to find the area of the region enclosed between the parabola $2y = x^2$ and the line $y = 2x$ (7)

- b) Find the volume of the solid in the first octant bounded by the coordinate planes and the plane $x + 2y + z = 6$. (7)

- 16 a) Change the order of integration and hence evaluate $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy dx$ (7)

- b) Evaluate $\iiint_G z dV$, where G is the wedge in the first octant cut off from the cylindrical solid $y^2 + z^2 \leq 1$ and the planes $y = x$ and $x = 0$. (7)

Module-IV

- 17 a) Test the convergence of the series

$$1 + \frac{1.3}{3!} + \frac{1.3.5}{5!} + \frac{1.3.5.7}{7!} + \dots \quad (7)$$

- b) Find the sum of the series
- $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$
- (7)

- 18 a) Test the convergence of (i)
- $\sum_{k=1}^{\infty} \frac{k!}{3!(k-1)!3^k}$
- (ii)
- $\sum_{k=1}^{\infty} \left(\frac{4k-5}{2k+1}\right)^k$
- (7)

- b) Test the absolute or conditional convergence of
- $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^3+1}$
- (7)

Module-V

- 19 a) Expand into a Fourier series,
- $f(x) = e^{-x}$
- ,
- $0 < x < 2\pi$
- (7)

- b) Find the half range cosine series for
- $f(x) = (x-1)^2$
- in
- $0 \leq x \leq 1$
- . (7)

- 20 a) Find the Fourier series of the function
- $f(x) = |x|$
- in
- $-1 \leq x \leq 1$
- (7)

- b) Find the Fourier sine series of
- $f(x) = x \cos x$
- in
- $0 < x < \pi$
- (7)
